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01 Thermal Physics

Thermal Properties of Matter

Temperature and Thermal Equilibrium

A branch of science which deals with the measurement of temperature of a substance is known as thermometry.

Temperature

Temperature is relative measure or indication of **degree of hotness** or **coldness** of a body.

It is a scalar physical quantity with S.I. unit Kelvin (K).

Example: Normal temperature of human body is 310. 15 K = 37°C = 98.6°F

NTP or STP implies 273.15K = 0°C = 32°F

Heat : When two bodies of different temperature are made in contact, the energy will flow from higher temperature body to lower temperature body, which is known as heat.

Note : (i) For heat transfer, there must be temperature difference between bodies in contact.

(ii) Heat is a energy in transit.

(iii) Unit : SI unit : joule (J) ; CGS :- erg ; Practical unit :- calorie

(iv) Unit Conversion : 1 Joule = 10^7 erg

1 calorie = 4.186 Joule ≈ 4.2 Joule

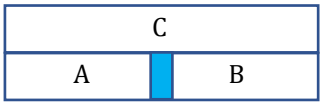
Thermal equilibrium

Two bodies are said to be in thermal equilibrium, when no (net) heat flows from one body to the other i.e. when both the bodies are at the same temperature.

If $T_A = T_B$, then net heat flow between them is zero.

Zeroth law of thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduces the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are same.



In measuring the temperature of a body, it is important that the thermometer should be in thermal equilibrium with the body whose temperature is to be measured.

Illustration 1:

If two bodies have same temperature but the amount of heat energy in both of them is different. Will there be a flow of heat energy between them if both bodies are placed in contact?

Solution:

As the temperature of the two bodies are same they are in thermal equilibrium which means there would be no transfer of heat energy between them. Flow of heat energy takes place when there is temperature difference between the bodies which are in contact with each other.

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Temperature Scale

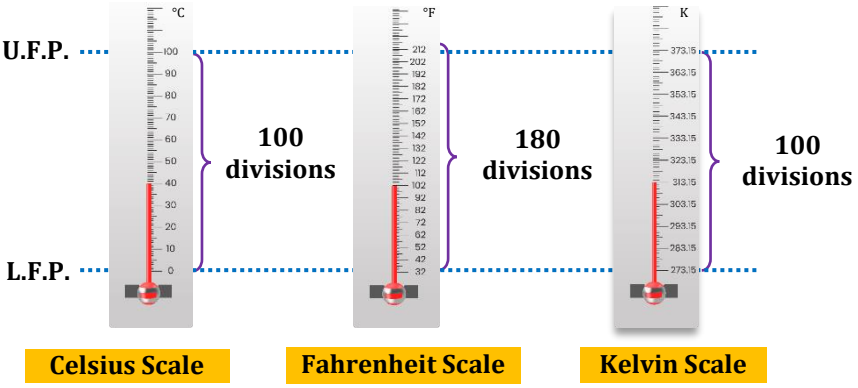
A thermometer is a device used to measure the temperature of a body.
Property of a material which changes with the change in temperature is called **Thermometric Property** like expansion of liquid or gas.

Different Scales of temperature

A thermometer can be graduated into following scales :

(a) The Centigrade or Celsius scale (°C)
(b) The Fahrenheit scale (°F)
(c) Kelvin scale (K)

Scale	Symbol	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Kelvin	K	273.15K	373.15K	100



Relation between different scale :

- Temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - \text{LFP}}{\text{UFP} - \text{LFP}} = \text{Constant for all scales}$$

- All these temperatures are related to each other by the following relationship

$$\frac{C-0}{100} = \frac{F-32}{212-32} = \frac{K-273}{373-273} = \frac{A-LFP}{UFP-LFP}$$

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{A-LFP}{UFP-LFP}$$

Note: (i) To calculate difference in scale

Difference in temperature: $\frac{\Delta T_{°H}}{\text{B.P.} - \text{M.P.}} = \frac{\Delta C}{100} = \frac{\Delta F}{180} = \frac{\Delta K}{100}$

$$\frac{\Delta C}{5} = \frac{\Delta F}{9} = \frac{\Delta K}{5}$$

(ii) The Celsius and Kelvin scales have different zero points but the same size degrees. Therefore, any temperature difference is the same on the Celsius and Kelvin scales.
$(T_2 - T_1)°C = (T_2 - T_1)\,K$ OR $\Delta T°C = \Delta TK$

(iii) Reading of one division $\propto \frac{1}{\text{Total number of divisions}}$

If number of divisions increases $\rightarrow$ Reading of one division decreases
Order of one division reading
$1°F < 1°K = 1°C$

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(iv) Value of temperatures in Celsius and Fahrenheit scale may be same.
Value of temperature in Fahrenheit and kelvin may be same. But Value of temperature in Celsius scale and kelvin scale can never be same.

$$\frac{C}{5}=\frac{K-273}{5} \Rightarrow K = °C + 273$$

Scale of Temperature

Graph between of temperature of a body in degree Celsius and degree Fahrenheit

$$\frac{C}{5}=\frac{F-32}{9}$$

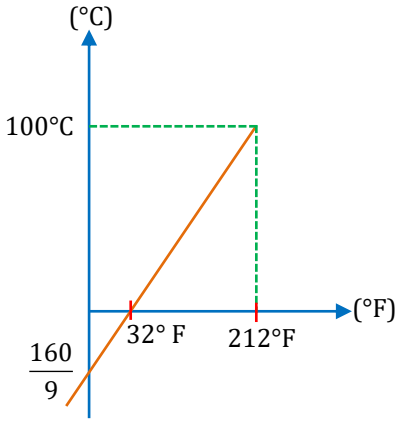
$$C=\frac{5}{9}F-\frac{160}{9} \quad ...(i)$$

$$y = mx + c \quad ...(ii)$$

From equations (i) and (ii)

$$\text{Slope} = m = \tan\theta = \frac{5}{9}$$

$$\text{y intercept} = c = \frac{160}{9}$$



Absolute Zero

Equals to **–273.15°C or 0 Kelvin**

At this temperature kinetic energy of gas molecules becomes zero.

Negative temperature is not possible in kelvin scale.

Illustration 2:

Temperature of a patient is 40°C. Find the temperature on Fahrenheit scale?

Solution:

$$\frac{F-32}{180}=\frac{40-0}{100}\Rightarrow F=104F$$

Illustration 3:

At what temperature is the Fahrenheit scale reading equal to twice of Celsius?

Solution:

$$\frac{F-32}{180}=\frac{C-0}{100}\Rightarrow\frac{2x-32}{180}=\frac{x-0}{100}\Rightarrow x=160$$

Illustration 4:

The lower and upper fixed points of a faulty thermometer are 5 W and 105 W. If the thermometer reads 25 W, what is the actual temperature in Celsius scale?

Solution:

$$\frac{25-5}{100}=\frac{C-0}{100}\Rightarrow C=20°C$$

Illustration 5:

A thermometer with an arbitrary scale has the ice point at –20° and the steam point at 180°. When the thermometer reads 5°, a Centigrade thermometer will read

Solution:

$$\frac{C-0}{100-0}=\frac{t-(-20)}{180-(-20)} \text{ (Here } t = 5°) \Rightarrow \frac{C}{100}=\frac{5+20}{200}\Rightarrow C=12.5C$$

Illustration 6:

The temperature of an iron piece is raised from 30°C to 90°C. What is the change in its temperature on the Fahrenheit scale and on the Kelvin scale?

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Solution:

$\Delta C = 90° - 30° = 60°C$

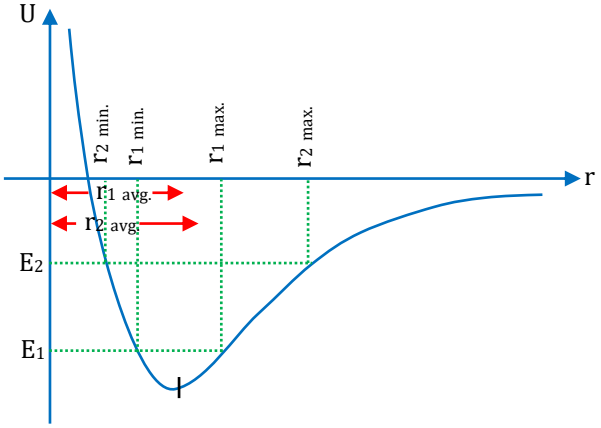
Temperature difference on Fahrenheit Scale $\Delta F = \frac{9}{5}\Delta C = \frac{9}{5}(60°C) = 108°F$

Temperature difference on Kelvin Scale $\Delta K = \Delta C = 60K$

Thermal Expansion

When matter is heated without any change in its state, it usually expands. According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion. As with rise in temperature the amplitude of vibration increases and hence energy of atoms increases, hence the average distance between the atom increases. So, the matter as a whole expands.

According to atomic theory of matter, asymmetry in potential energy curve is responsible for thermal expansion as with rise in temperature say from T_1 to T_2 the amplitude of vibration and hence energy of atoms increases from E_1 to E_2 and hence the average distance between the from r_1 to r_2. Due to this increase in distance between atoms, the matter as a whole expands which is known as thermal expansion. Had the potential energy curve been symmetrical, no thermal expansion would have taken place in spite of heating



- Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular force is maximum in solids but minimum in gases.
- Solids can expand in one dimension (linear expansion), two dimension (superficial expansion) and three dimensions (volume expansion) while liquids and gases usually suffers change in volume only

Linear Expansion:

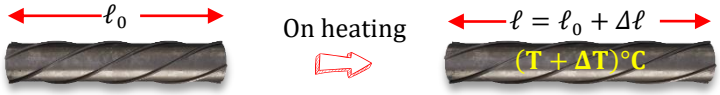
When a solid is heated and its length increases, then the expansion is called linear expansion.

$d\ell \propto dT$ and $d\ell \propto \ell_0$

$d\ell = \alpha \ell_0 dT$

$\int d\ell = \int \alpha \ell_0 dT$

If α is constant



- Change in length $\Delta\ell = \alpha \ell_0 \Delta T$

  (ℓ_0 = Original length, ΔT = Temperature change)

- Co-efficient of linear expansion, $\alpha = \frac{\Delta\ell}{\ell_0 \Delta T}$

  If $\ell_0 = 1$ and $\Delta T = 1°C$; Then, $\alpha = \Delta\ell$

- $\ell - \ell_0 = \alpha \ell_0 \Delta T$

  Final length $\ell = \ell_0 + \alpha \ell_0 \Delta T$

- Unit of α is $°C^{-1}$ or K^{-1}. It's dimension is $[\theta^{-1}]$

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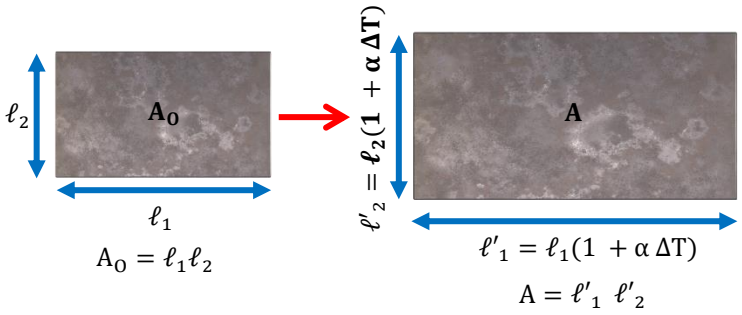
Superficial (areal) expansion:

When the temperature of a 2D object is changed, it's area changes, then the expansion is called superficial expansion.

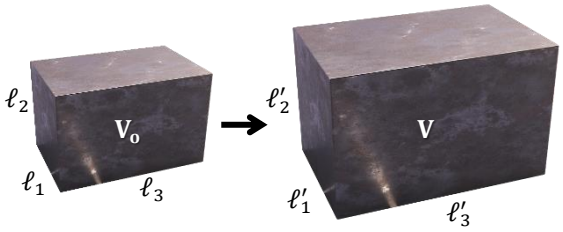


$$A = \ell_1(1+\alpha\Delta T)\times\ell_2(1+\alpha\Delta T)$$
$$A = \ell_1\ell_2(1+\alpha\Delta T)^2$$
$$A = A_0(1+2\alpha\Delta T)$$
$$A = A_0 + A_0\beta\Delta T \qquad \because \beta = 2\alpha$$

- Change in area is $\Delta A = A_0\beta\Delta T$
  (A_0 = Original area, ΔT = Temperature change)
- Co-efficient of superficial expansion $\beta = \dfrac{\Delta A}{A_0\Delta T}$
- Final area $A = A_0(1+\beta\Delta T)$
- Unit of β is °C^{-1} or K^{-1}.

Volume or cubical expansion:

When a solid is heated and it's volume increases, then the expansion is called volume or cubical expansion.



$$V_0 = \ell_1\times\ell_2\times\ell_3$$
$$V = \ell_1\ell_2\ell_3(1+\alpha\Delta T)^3$$
$$V = V_0(1+3\alpha\Delta T) \qquad \because \gamma = 3\alpha$$
$$V = V_0(1+\gamma\Delta T)$$
$$\Delta V = V_0\gamma\Delta T$$

- Change in volume is $\Delta V = V_0\gamma\Delta T$
  (V_0 = Original volume, ΔT = change in temperature)
- Final volume $V = V_0(1+\gamma\Delta T)$
- Volume co-efficient of expansion $\gamma = \dfrac{\Delta V}{V_0\Delta T}$
- Unit of γ is °C^{-1} or K^{-1}.

Note : The co-efficient α, β and γ for a solid are related to each other as follows

For isotropic material, $\alpha_x = \alpha_y = \alpha_z = \alpha$

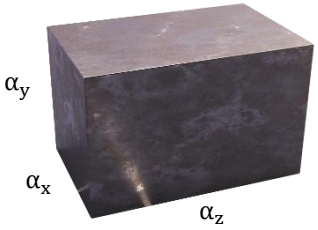
→ expansion coefficient for isotropic material

$\beta = 2\alpha$ $\qquad$ $\gamma = 3\alpha$

$= \alpha:\beta:\gamma = 1:2:3$

→ coefficient of expansion for anisotropic material

$\gamma = \alpha_x + \alpha_y + \alpha_z$ $\qquad$ $\beta = \alpha_x + \alpha_y$



$\alpha_x \rightarrow$ coefficient of linear expansion

$\alpha_y \rightarrow$ coefficient of superficial expansion, $\alpha_z \rightarrow$ coefficient of cubical expansion

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Illustration 7:

Iron rod of 50 cm is heated from 20°C to 100°C. Find length of rod at 100°C if $\alpha_{iron} = 12\times10^{-6}\ °C^{-1}$

Solution:

$\ell_2 = \ell_1[1+\alpha(T_2-T_1)] \Rightarrow \ell_2 = 50[1+12\times10^{-6}\times80]$

$\ell_2 = 50[1+96\times10^{-5}] = 50+48\times10^{-3} \Rightarrow \ell_2 = 50.048\text{cm}$

Illustration 8:

A metal rod of silver at 0°C is heated to 100°C. It's length is increased by 0.19 cm. Coefficient of cubical expansion of the silver rod is

Solution:

$$\alpha = \frac{\Delta L}{L_0(\Delta\theta)} = \frac{0.19}{100(100-0)} = 1.9\times10^{-5}/°C \Rightarrow \gamma = 3\alpha = 3\times1.9\times10^{-5}/°C = 5.7\times10^{-5}/°C$$

Illustration 9:

If volume of metal increases by 0.12% on increasing temperature by 20°C. Find α?

Solution:

$$\gamma = \frac{\Delta V}{V\Delta T} \Rightarrow \frac{\Delta V}{V} = \frac{0.12}{100} = 12\times10^{-4}$$

$$\therefore \gamma = 3\alpha = \frac{12\times10^{-4}}{20} \Rightarrow \alpha = \frac{1}{5}\times10^{-4} \Rightarrow \alpha = 0.2\times10^{-4}$$

Illustration 10:

The difference between lengths of a certain brass rod and of a steel rod is claimed to be constant at all temperatures. Is this possible?

Solution:

If L_B and L_S are the lengths of brass and steel rods respectively at a given temperature, then the lengths of the rods when temperature is changed by θ °C.

$$L_B' = L_B(1+\alpha_B\Delta\theta) \text{ and } L_S' = L_S(1+\alpha_S\Delta\theta)$$

So that, $$L_B' = L_S'(L_B - L_S) + (L_B\alpha_B - L_S\alpha_S)\Delta\theta$$

So $(L_B' - L_S')$ will be equal to $(L_B - L_S)$ at all temperatures if, $L_B\alpha_B - L_S\alpha_S = 0$ [as $\Delta\theta \neq 0$]

or $$\frac{L_B}{L_S} = \frac{\alpha_S}{\alpha_B}$$

i.e., the difference in the lengths of the two rods will be independent of temperature if the lengths are in the inverse ratio of their coefficients of linear expansion.

Illustration 11:

A composite bar of length $L = L_1 + L_2$ is made up from a rod of material 1 and of length L_1 attached to a rod of material 2 and of length L_2 as shown. If a_1 and a_2 are their respective coefficients of linear expansion, then equivalent coefficient of linear expansion for the composite rod is :

1	2

Solution:

When it has been given a temperature change ΔT, the new length of the composite bar is the sum of the individual final lengths (Let ' α ' be required coefficient of linear expansion)

$$L' = L'_1 + L'_2$$

$$L(1+\alpha\Delta T) = L_1(1+\alpha_1\Delta T) + L_2(1+\alpha_2\Delta T)$$

$$\Rightarrow \quad L + L\alpha\Delta T = L_1 + \alpha_1\Delta T + L_2 + L_2\alpha_2\Delta T$$

$$\Rightarrow \quad L + L\alpha\Delta T = L + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T \qquad (\because L_1 + L_2 = L)$$

$$\Rightarrow \quad L\alpha\Delta T = L_1\alpha_1\Delta T + L_2\alpha_2\Delta T \qquad \Rightarrow \qquad \alpha = \frac{L_1\alpha_1 + L_2\alpha_2}{L}$$

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Illustration 12:

The co-efficient of linear expansion of crystal is one direction is α_1 and other two directions is α_2 then co-efficient of volume expansion will be: -

Solution:

We know coefficient of expansion for anisotropic material

$\gamma = \alpha_x + \alpha_y + \alpha_z \qquad \therefore \gamma = \alpha_1 + \alpha_2 + \alpha_2 \qquad \Rightarrow \gamma = \alpha_1 + 2\alpha_2$

Illustration 13:

If I is the moment of inertia of a solid body having α - coefficient of linear expansion then the change in I corresponding to a small change in temperature ΔT is -

Solution:

Moment of inertia $I = MR^2$

or $\Rightarrow \dfrac{dI}{dR} = 2MR$

or $\Delta I = 2MR.\Delta R \qquad ...(i)$

By the definition of linear co-efficient of expansion

$\alpha = \dfrac{R - R_0}{R.\Delta T} = \dfrac{\Delta R}{R\Delta T}$

from equation (i)

$\Delta I = 2MR.\alpha R\Delta T$

or $\Delta I = 2I\alpha\Delta T$

Applications of Thermal Expansion

Change in time period of simple pendulum

A pendulum clock keeps proper time at temperature θ. If temperature is increased to $\theta'(>\theta)$ then due to linear expansion, length of pendulum increases and hence its time period will increase.

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad \Rightarrow \qquad \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell}$$

Fractional change in time period = $\dfrac{\Delta T}{T} = \dfrac{1}{2}\alpha\Delta\theta$

Change in time = $\Delta T = \dfrac{1}{2}\alpha\Delta\theta T$ ($\Delta\theta \rightarrow$ Change in temperature)

- The clock will lose time i.e. will become slow in summer and will gain time i.e. will become fast in winter

In summer	In winter
⬇	⬇
Temperature $\Delta\theta$ increases	Temperature $\Delta\theta$ decreases
⬇	⬇
Length of pendulum increases	Length of pendulum decreases
⬇	⬇
Time period will increase	Time period will decrease
⬇	⬇
Clock will become slow	Clock will become fast
⬇	⬇
Loss of time $\Delta T = \dfrac{1}{2}\alpha\Delta\theta T$	Gain of time $\Delta T = \dfrac{1}{2}\alpha\Delta\theta T$

- Since coefficient of linear expansion (α) is very small for invar, hence pendulums are made of invar to show the correct time in all seasons.

Note : Time loss/Gain in a day, $\Delta T = \frac{1}{2}\alpha\Delta\theta \times (86400\,\text{second})$

Illustration 14:

A pendulum clock consists of an iron rod connected to a small heavy bob. If it is designed to keep the correct time at 20 °C, how fast or slow will it go in 24 hours at 40 °C? ($\alpha_{iron} = 1.2 \times 10^{-6}\,°C^{-1}$)

Solution:

Given,

Since the length of the pendulum increases due to thermal expansion, the time period will also increase.

Gain in time period per second $= \frac{1}{2}\alpha\Delta\theta$

Therefore, the amount by which the time gets slowed in 24 hours is,

$$\Delta T = \frac{1}{2}\alpha\Delta\theta \times 24 \times 60 \times 60$$

or $\Delta T = \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 24 \times 60 \times 60$ or $\Delta T = 1.0368\ s$

Hence, due to an increase in the length of the pendulum, time gets slowed down by 1.0368 s in a day.

Expansion of Isotropic material:

Expansion of cavity: Thermal expansion of an isotropic object may be imagined as a photographic enlargement. So, if there is a hole A in a plate C (or cavity A inside a body C), the area of hole (or volume of cavity) will increase when body expands on heating, just as if the hole (or cavity) were solid B of the same material. Also, the expansion of area (or volume) of the body C will be independent of shape and size of hole (or cavity), i.e., will be equal to that of D.

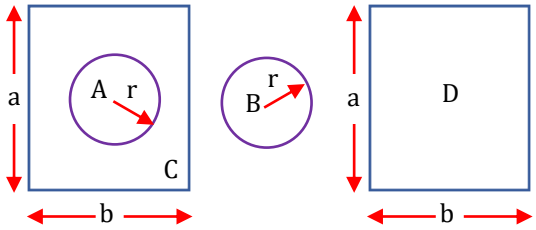
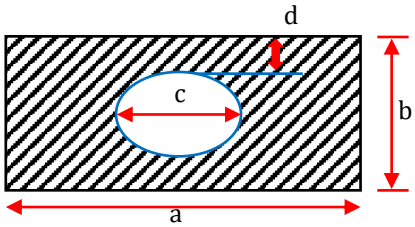


Illustration 15:

A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.

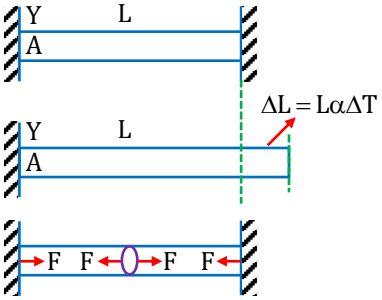


Solution:

Distance between any two point on an object increases with increase in temperature. So, all dimension a, b, c and d will increase

Thermal stress in a rigidly fixed rod

When a rod whose ends are rigidly **fixed** such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports.

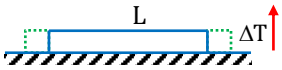


Thermal strain $= \frac{\Delta L}{L} = \alpha \Delta T$ $\left[\text{As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta T}\right]$

So Thermal stress = $Y\alpha\Delta T$ $\left[\text{As } Y = \frac{\text{Stress}}{\text{strain}}\right]$ Here Y is Young's Modulus

or Force on the supports $F = YA\alpha\Delta T$

Free Expansion



If rod is not fixed at end i.e. free to expand there will be zero stress.

Illustration 16:

A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5}/°C and its Young's modulus is 2.0×10^{11} N/m^2.

Solution:

Here final length is less than the original length so that strain is tensile and tensile force is given by

$F = AY\alpha\Delta\theta = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20 = 24$ N

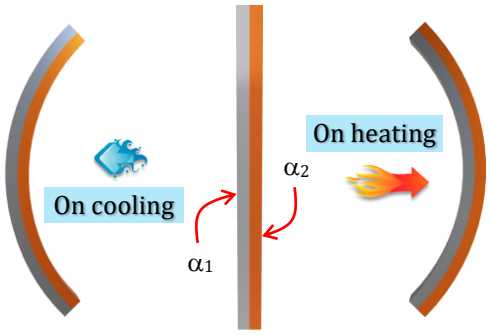
Bimetallic Strip

When two strips of equal length but of different materials (different coefficient of linear expansion) are joined together, it is called "Bi–metallic strip" and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metals. The strip will bend with metal of greater α on outer side. Coefficient of expansion is more for brass than steel

Two strips of equal lengths but of different coefficient of linear expansion.

If $\alpha_2 > \alpha_1$

On heating the strip will bend with metal of greater a on outer side i.e. convex side.

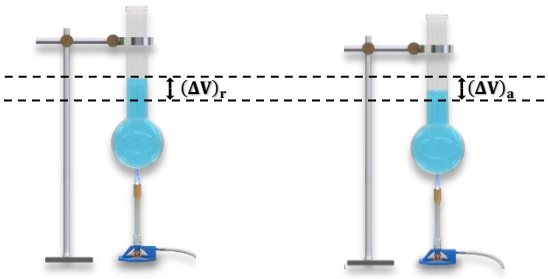


Thermal Expansion in Liquids

- Liquids do not have linear and superficial expansion but these only have volumetric expansion
- Since liquids are always to be heated along with a vessel which contains them so initially on heating the system (liquid + vessel), the level of liquid in vessel falls (as vessel expands more since it absorbs heat and liquid expands less) but later on, it starts rising due to faster expansion of the liquid.

  QP $\rightarrow$ represents expansion of vessel .

  QR $\rightarrow$ represents the real expansion of liquid.
- The actual increase in the volume of the liquid = The apparent increase in the volume of liquid + the increase in the volume of the vessel.



Liquids have two coefficients of volume expansion

1. Co-efficient of apparent expansion (γ_a) :

It is due to apparent (that appears to be, but is not) increase in the volume of liquid if expansion of vessel containing the liquid is not taken into account.

$$\gamma_a = \frac{\text{Apparent expansion in volume}}{\text{initial volume} \times \Delta T} = \frac{(\Delta V)_a}{V \times \Delta T}$$

2. Co-efficient of real expansion (γ_r) :

It is due to the actual increase in volume of liquid due to heating.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{initial volume} \times \Delta T} = \frac{(\Delta V)_r}{V \times \Delta T}$$

- Also, coefficient of expansion of flask $\gamma_{vessel} = \frac{(\Delta V)_{Vessel}}{V \times \Delta T}$
- Liquids do not have linear and superficial expansion but these only have volume expansion.

  The apparent increase in the volume of liquid = The actual increase in the volume of the liquid – The increase in the volume of the vessel

  $\Delta V_{apparent} = \Delta V_{real} - \Delta V_{vessel}$

  $\gamma_{app} = \gamma_{real} - \gamma_{vessel}$

  $\gamma_{real} = \gamma_{app} + \gamma_{vessel}$
- Change (apparent change) in volume in liquid relative to vessel is

  $\Delta V_{app} = V\,\gamma_{app}\,\Delta T = V(\gamma_{Real} - \gamma_{Vessel})\Delta T \qquad (\because \gamma_{app} = \gamma_{real} - 3\alpha)$

  $\Delta V_{app} = V(\gamma_{real} - 3\alpha)\Delta T$

  α = Coefficient of linear expansion of the vessel.

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Different level of liquid in vessel

γ	ΔV	Level
$\gamma_{Real.} > \gamma_{Vessel}(=3\alpha) \Rightarrow \gamma_{app} > 0$	$\Delta V_{app.}$ is positive	Level of liquid in vessel will rise on heating.
$\gamma_{Real.} < \gamma_{Vessel}(=3\alpha) \Rightarrow \gamma_{app} < 0$	$\Delta V_{app.}$ is negative	Level of liquid in vessel will fall on heating.
$\gamma_{Real.} = \gamma_{Vessel}(=3\alpha) \Rightarrow \gamma_{app} = 0$	$\Delta V_{app.}$ is zero	Level of liquid in vessel will remain same.

Change in Density

$$\rho_0 = \frac{M}{v_0}$$

$$\rho_0 = \frac{M}{v} = \frac{M}{v_0(1+\gamma\Delta t)}$$

$$\rho = \rho_0(1+\gamma\Delta t)^{-1}$$

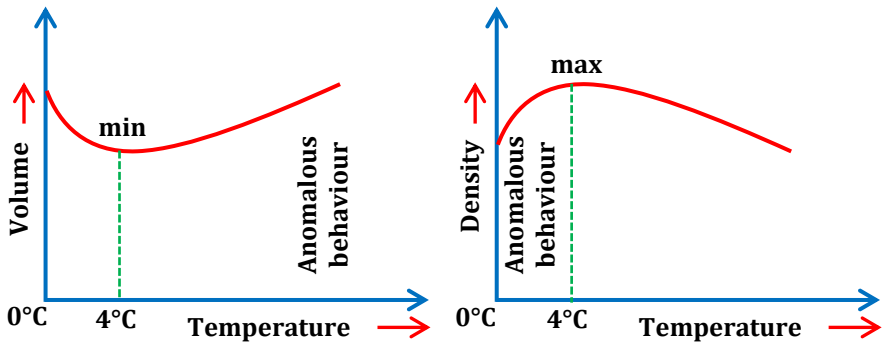
$$\rho = \rho_0(1-\gamma\Delta t) \qquad \text{(By binomial expension)}$$

$$\frac{\Delta\rho}{\rho} = -\gamma\Delta t \qquad \text{(–ve sign as on heating density decreases)}$$

Anomalous behaviour of water

Generally, matter expands on heating and contracts on cooling. In case of water, it expands on heating if its temperature is greater than 4°C. In the range 0°C to 4°C, water contracts on heating and expands on cooling, i.e. γ is negative. This behaviour of water in the range from 0°C to 4°C is called anomalous expansion.

This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature increases initially at the surface. The water there sinks because of its increased density. Consequently, the surface reaches 0°C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4°C.



- At 4°C, density of water is maximum while its specific volume is minimum

Some other application

- When rails are laid down on the ground, space is left between the ends of two rails.
- The transmission cable is not tightly fixed to the poles.
- Test tubes, beakers and crucibles are made of Pyrex-glass or silica because they have very low value of coefficient of linear expansion.
- The iron rim to be put on a cart wheel is always of slightly smaller diameter than that of wheel.
- A glass stopper jammed in the neck of a glass bottle can be taken out by warming the neck of the bottle

Illustration 17:

A glass vessel of volume 100 cm^3 is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8×10^{-6}/°C and coefficient of volume expansion of mercury is 1.8×10^{-4}/°C.

Solution:

$$\Delta V = V_0(\gamma_L - \gamma_C)\Delta T = 100\left\{1.8\times10^{-4} - 3\times1.8\times10^{-6}\right\}\times50 \Rightarrow \Delta V = 0.87 cm^3$$

Illustration 18:

A glass flask of volume 4 litre is fully filled with mercury at 0°C. Both the flask and mercury are now heated to 25°C. If the coefficient of volume expansion of mercury is $1.82 \times 10^{-4}\ °C^{-1}$ and coefficient of linear expansion of glass is $1 \times 10^{-5}\ °C^{-1}$, the amount of mercury which is spilled out is–

Solution:

$\Delta V = V\gamma_a\Delta t = V(\gamma_r - 3\alpha_g)\Delta t = 4 \times 1000\ (1.82 \times 10^{-4} - 3 \times 1 \times 10^{-5}) \times 25 = 15.2$ ml.

Illustration 19:

A liquid with coefficient of volume expansion γ is filled in a container of a material having the coefficient of linear expansion α. If the liquid overflows on heating, then –

Solution:

Volumetric coefficient of expansion of material is 3α.

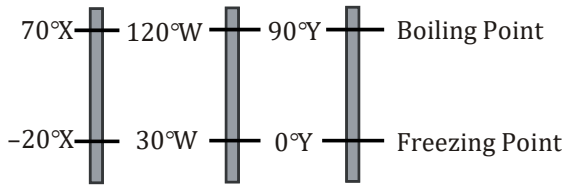
As the system is heated liquid started flowing out of box, it means volumetric coefficient of expansion of liquid is greater than of the container.

So $\gamma > 3\alpha$

BEGINNER'S BOX-1

1. Write down the following temperatures in the increasing order 50°F, 50°C and 50 K.

2. The figure shows three temperature scales with the freezing and boiling points of water indicated.



(a) Rank the size of a degree on these scales, greatest first.

(b) Rank the following temperatures, highest first : 50°X, 50°W and 50°Y.

3. What is the temperature at which we get the same reading on both the Centigrade and Fahrenheit scales?

4. A thin copper wire of length L increases in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2?

(A) 1% (B) 2% (C) 3% (D) 4%

5. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C ?

Coefficient of linear expansion of copper = $1.70 \times 10^{-5}\ °C^{-1}$,

6. If a bimetallic strip is heated, it will

(A) bend towards the metal with lower thermal expansion coefficient.

(B) bend towards the metal with higher thermal expansion coefficient.

(C) twist itself into helix.

(D) have no bending.

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Heat (Q)

When Two bodies at different temperature are made in contact, then net energy will flow from higher temperature body to lower temperature body, which is known as **'Heat'.**

Calorie

The amount of heat needed to increase the temperature of 1g of water form 14.5°C to 15.5°C at a pressure of 1 atm is called 1 calorie.

1 calorie = 4.186 Joule $\approx$ 4.2 Joule

Mechanical Equivalent of Heat

According to Joule, work may be converted into heat and vice-versa. The ratio of work done to the heat produced is always constant.

$\frac{W}{Q}$ = constant = J $\Rightarrow$ W = J Q

W must be in joule and Q must be in calorie.

J is called mechanical equivalent of heat. It is not a physical quantity but simply a conversion factor.

J = 4.18 $\simeq$ 4.2 joule/Cal = 4.186×10^3 joule/kcal (J is dimensionless)

For rough calculations we take J = 4.2 joule/cal

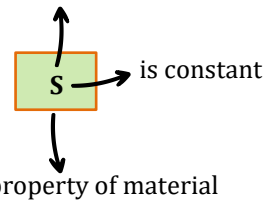
$Q \propto m\Delta T \qquad \Rightarrow \qquad Q = mS\Delta T$

Specific Heat (S or C)

Amount of heat required to change the temperature of unit mass of substance by 1°C or 1K is called specific heat.

$$\therefore S = \frac{Q}{m\Delta T}$$

Does not depend on shape and mass of the body



Where, Q = Heat

S = Specific Heat capacity

m = mass of substance

ΔT = Change in temperature

The specific heat depends on the pressure, volume and temperature of the substance.

For liquids and solids, specific heat measurements are most often made at a constant pressure as functions of temperature, because constant pressure is quite easy to produce experimentally.

- Unit : SI :- joule/kg–K ; CGS :- cal/gm –°C
- Specific heat of water : S_{water} = 1 cal/gm–°C = 1 cal/gm–K = 1 kcal/kg–K = 4200 joule/kg–K
- Specific heat capacity of Ice/steam : $S_{ice/steam} = \frac{1}{2}\frac{cal}{gm°C} = \frac{4.2J}{2\times10^{-3}kg°C}$ = 2100 J/KgK.
- When a substance does not undergo a change of state (i.e., liquid remains liquid or solid remains solid), then the amount of heat required to raise the temperature of mass m of the substance by an amount ΔT is,

  $Q = mS\Delta T$
- These specific heats can be molar or gram.

Molar Specific Heat (S_{mol})

The amount of energy needed to raise the temperature of one mole of a substance by 1°C (or 1K) is called molar heat capacity. The molar heat capacity is the product of molecular weight and specific heat i.e.,

If the molecular mass of the substance is M and the mass of the substance is m then number of moles of the substance

$$S_{mol} = \frac{M}{m}\left(\frac{Q}{\Delta T}\right) \qquad \left[\because \mu = \frac{m}{M}\right]$$

$$S_{mol} = \frac{Q}{\mu \Delta T}$$

if $\mu = 1$ and $\Delta T = 1 \quad \Rightarrow \quad S_{mol} = Q$

- Unit : SI $\rightarrow \frac{J}{molK}$ or $\frac{J}{mol°C}$; CGS $\rightarrow \frac{cal}{mol°C}$

Gram Specific Heat (S_{gram})

The amount of energy needed to raise the temperature of one gram of a substance by 1°C (or 1K) is called gram heat capacity.

$$S_{gram} = \frac{Q}{m\Delta T}$$

if $m = 1gram$ and $\Delta T = 1$

$S_{gram} = Q$

- Unit : SI $\rightarrow \frac{J}{gramK}$ or $\frac{J}{gram°C}$, CGS $\rightarrow \frac{cal}{gram°C}$

Relation between Gram and Molar Specific heat

$$Q = mS_{gram}\Delta T = \mu S_{mol}\Delta T$$

$$mS_{gram} = \mu S_{mol}$$

$$mS_{gram} = \frac{m}{M_w}S_{mol}$$

$$S_{mol} = M_w S_{gram}$$

Note : Specific heat of solid and liquids are considered to be constant.
In all-natural substances specific heat capacity of H_2 gas is maximum.
In remaining solid and liquid, it is maximum for water.
Due to high specific heat water can be used as a good coolant but its boiling point is low.

Heat Capacity

Heat capacity of a body is defined as the amount of heat required to raise the temperature of a **given mass** of a body by 1°C or 1K. It is also called thermal capacity.

$$\text{Heat capacity} = \frac{Q}{\Delta T}$$

Heat capacity = mS = mass × specific heat

- Unit : SI $\rightarrow \frac{J}{K}$ or JK^{-1} or $J°C^{-1}$; Dimensions : $ML^2T^{-2}K^{-1}$

Note: Thermal capacity of a body depends upon mass of the substance, nature & geometry of the substance.

Latent Heat / Hidden Heat

Heat required to change the state of unit mass depends on the nature of substance and is called latent heat of substance.

$L = \frac{Q}{m} \Rightarrow Q = mL$

When phase of a body changes, change of phase takes place at constant temperature [melting point or boiling point] and heat released or absorbed is Q = mL where L is latent heat. Heat is absorbed if solid converts into liquid (at melting point) or liquid convert into vapours (at boiling point) and heat is released if liquid converts into solid or vapours convert into liquid.

- Phase Conversion : During the phase change, there will be no change in temperature.
  Whatever heat you provide, will go in changing the phase of the substance.

  $\text{Solid} \underset{\text{freezing}}{\overset{\text{Melting}}{\rightleftharpoons}} \text{liquid}$

  $\text{Liquid} \underset{\text{condensation}}{\overset{\text{Vaporization}}{\rightleftharpoons}} \text{Gas}$

  Unit : $SI \rightarrow J/kg$; $CGS \rightarrow cal/gm$

Latent heat of fusion

It is the quantity of heat (in kilocalories) required to change 1 kg mass of a substance from solid to liquid state at its melting point.

Latent heat of fusion for ice : (L_f) ice = 80 cal/g = 80 kcal/kg

Latent heat of fusion for water : (L_f) water = 80 cal/g = 80 kcal/kg

$[Q = mL_f]$

Latent heat of vaporization

The quantity of heat required to change its 1 kg mass from liquid to vapour state at its boiling point.

Latent heat of vaporization for water : 536 kcal/kg = 536 cal/g; $\simeq$ 540 cal/g

(L_V) water = 540 cal/g

$Q = mL_V \qquad \Rightarrow \qquad \left(L_V = \frac{Q}{m}\right)$

Note :

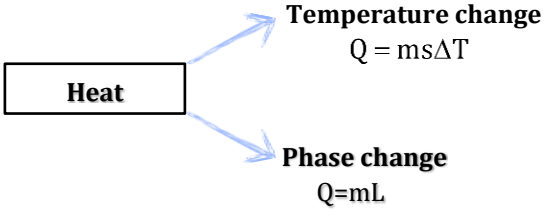


Illustration 20:

Find amount of heat required to convert 5 gram of ice from −10° C to steam at 130°C.

Solution:

$$\underset{-10^\circ C}{\text{Ice}} \rightarrow \underset{0^\circ C}{\text{Ice}} \rightarrow \underset{0^\circ C}{\text{Water}} \rightarrow \underset{100^\circ C}{\text{Water}} \rightarrow \underset{100^\circ C}{\text{Steam}} \rightarrow \underset{130^\circ C}{\text{Steam}}$$

$$\downarrow Q_1 \quad \downarrow Q_2 \quad \downarrow Q_3 \quad \downarrow Q_4 \quad \downarrow Q_5$$

$Q = ms_i\Delta T_1 + mL_f + ms_w\Delta T_2 + mL_v + ms_s\Delta T_2$

$Q = m(s_i\Delta T + L_f + s_w\Delta T_2 + L_v + s_s\Delta T_2)$

$Q = 5\left(\frac{1}{2}(10) + 80 + 1(100) + 540 + \frac{1}{2}(30)\right) = 3700 \text{ cal}$

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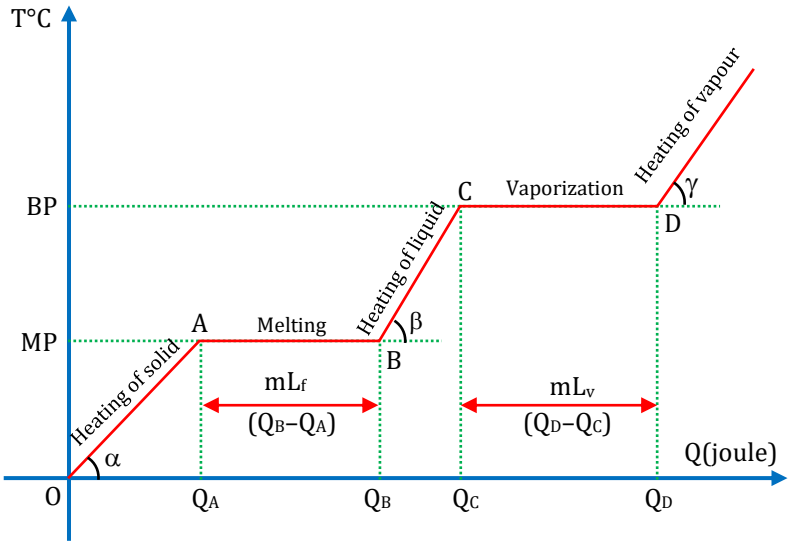
Heating Curve

Heating of Solid

If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and heat, as shown in figure, it is called heating curve.

Assumptions for heating curve

1. Initially object is in solid state and its temperature is less then melting point
2. Heat is given to the system by a constant power source.



$P = \frac{Q}{t}$ = constant

$\text{Slope} = \frac{dy}{dx} = \frac{dT}{dQ}$

$dQ = msdT$, $\frac{dT}{dQ} = \frac{1}{ms} = \text{slope} \Rightarrow \therefore ms = \frac{1}{\text{slope}}$

In heating curve inverse of slope give heat capacity (ms).

- In the region OA, BC, DE, Temperature of liquid increases so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line.
- Order of Slope $\alpha > \beta > \gamma$
- $(\text{Heating capacity})_{\text{vapour}} > (\text{Heating capacity})_{\text{liquid}} > (\text{Heating capacity})_{\text{solid}}$
- Latent heat $\propto$ Length of graph parallel to x-axis

  $L_F \propto$ Length of line ab ; $L_V \propto$ Length of line cd
- Latent heat of fusion is proportional to the length of line (AB) of zero slope. [In this region-specific heat $\propto \frac{1}{\tan 0^\circ} = \infty$]

  Latent heat of vaporisation is proportional to the length of line (CD) of zero slope. [In this region-specific heat $\propto \frac{1}{\tan 0^\circ} = \infty$]
- If to a given mass (m) of a solid, heat is supplied at constant rate and a graph is plotted between temperature and time.

  $Q = ms\Delta T \Rightarrow P\Delta t = ms\Delta T$ $[\because Q = P\Delta t]$

  $\frac{\Delta T}{\Delta t} = \text{the slope of temperature – time curve} = \frac{1}{ms}$

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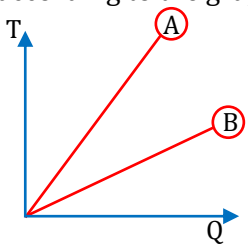
Illustration 21:

If substance A and B have same mass then according to the graph, find whose specific heat is greater?



Solution:

$\alpha_A > \alpha_\beta$

$\tan(\alpha_A) > \tan(\alpha_B)$

$(\text{slope})_A > (\text{slope})_B$

$s \propto \frac{1}{\text{slope}}$

$(s_A) < (s_B)$

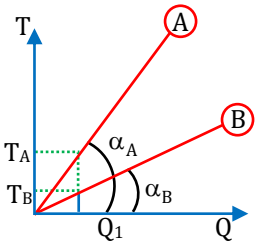


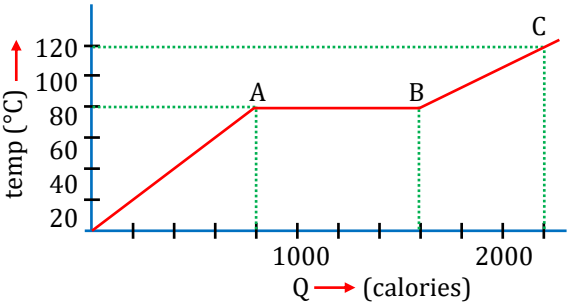
Illustration 22:

A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph

(i) the mass of the substance;

(ii) the specific latent heat of the melting process, and

(iii) the specific heat of the substance in the liquid state.



Solution:

(i) 0 to A (Heating to solid)

we know, $dQ = msdT$

$\Rightarrow \frac{dT}{dQ} =$ slope of T – Q graph $= \frac{1}{ms}$

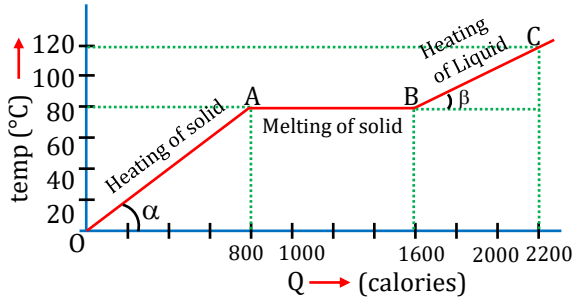
$\therefore$ For line OA

$\text{Slope} = \tan\alpha = \frac{80}{800} = \frac{1}{ms_{(solid)}}$

$ms_{(solid)} = 10$

m × 0.5 = 10

m = 20 gm = 0.02 kg



(ii) A to B (melting of solid)

Heat given during process A to B

$\Delta Q = 1600 - 800 = 800$ cal

$\Delta Q = 800 = mL_f \Rightarrow 800_{cal} = (0.02kg)L_f \Rightarrow L_f = 40{,}000$ cal/kg

(iii) (Heating of liquid)

$\tan\beta = \frac{dT}{dQ} = \frac{1}{ms_{(liq)}}$

$\frac{40}{600} = \frac{1}{(ms)_{liq}} \Rightarrow (ms)_{liq} = 15$

mass remains same

$(0.02kg)(s) = 15 \Rightarrow s = 750$ cal/kg K

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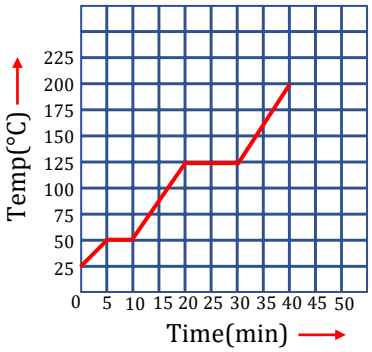
Illustration 23:

The graph shown in the figure represent change in the temperature of 5 kg of a substance as it absorbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vaporization of the substance is :



Solution:

Liquid to gas conversion takes place in between [20 min to 30 min]

Total time = 10 min

So that given = $\Delta Q = \left(42\times10^3\right)(10) = 42\times10^4\,J$

$\Delta Q = mL_F$

$L_f = \dfrac{42\times10^4\,J}{5kg} = 84Kj/kg$

Calorimetry

Principle of calorimetry

When two bodies at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

Heat Lost = Heat Gained (Principle of calorimetry)

Principle of calorimetry represents the law of conservation of heat energy.

Law of Mixtures

Heat loss = Heat gain

If 2 substance at different temperature are mixed in a calorimeter then heat given by substance at high temperature must be equal to heat absorbed by substance at lower temperature.

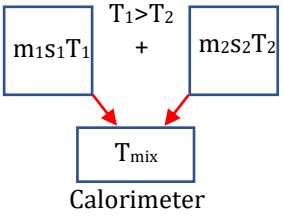
Heat given = heat absorbed

$m_1s_1\Delta T_1 = m_2s_2\Delta T_2$

$m_1s_1\,(T_1 - T_{mix}) = m_2s_2\,(T_{mix} - T_2)$

(s = specific heat)

$(T_1 > T_{mix} > T_2)$



Temperature of mixture (T) is always ≥ lower temperature (T_L) and ≤ higher temperature (T_H)

$$T_L \le T \le T_H$$

The temperature of mixture can never be lesser than lower temperature (as a body cannot be cooled below the temperature of cooling body) and greater than higher temperature (as a body cannot be heated above the temperature of heating body). Furthermore usually, rise in temperature of one body may not be equal to the fall in temperature of the other body though heat gained by one body is equal to the heat lost by the other.

Water Equivalent

The water equivalent of a body is the amount of water (W in gram) that absorbs or gives out the same amount of heat as is done by the body when heated or cooled through same temperature

H_C substance = H_C water

$M_sS_s = M_wS_w$

M_sS_s = W×1 cal/g°C

Water Equivalent = W in gram

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- Numerically water equivalent and heat capacity are same but there Unit is different.

X cal/°c	X g
Y J/°c or $\frac{Y}{4.2}$ cal/°c	$\frac{Y}{4.2}$ g
Z kcal/ °c	Z kg

Note : whenever water equivalent or heat capacity of container is given in that case heat consumed by container will also be calculated.

Illustration 24:

Liquid A(m_1,s_1,T_1) is mixed with liquid B (m_2,s_2,T_2). Find the temperature of mixture.

Solution:

$T_1 > T_2$

$m_1s_1 (T_1-T_{mix}) = m_2s_2 (T_{mix} - T_2)$

$m_1s_1T_1 - m_1s_1T_{mix} = m_2s_2T_{mix} - m_2s_2T_2$

$m_1s_1T_1 + m_2s_2T_2 = m_1s_1T_{mix} + m_2s_2T_{mix}$

$$\frac{m_1s_1T_1 + m_2s_2T_2}{m_1s_1 + m_2s_2} = T_{mix}$$

If $s_1 = s_2$, then $T_{mix} = \frac{m_1T_1 + m_2T_2}{m_1 + m_2}$

Illustration 25:

A calorimeter of heat capacity 100 J/K is at room temperature of 30°C. 100 g of water at 40°C of specific heat 4200 J/kg–K is poured into the calorimeter. What is the temperature of water in calorimeter?

Solution:

Let the temperature of water in calorimeter is t. Then heat lost by water = heat gained by calorimeter

$(0.1) \times 4200 \times (40 - t) = 100 (t - 30) \Rightarrow 420 \times 40 - 420\, t = 100\, t - 3000 \Rightarrow t = 38.07$ °C

Illustration 26:

5 g of ice at 0°C is kept in a calorimeter of water equivalent 20 g. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat)

Solution:

5 g of ice at 0°C is kept in a calorimeter of water equivalent 20 g

0^oC CALORIMETER $\xrightarrow[Ms\Delta T]{}$ CALORIMETER 100^oC

0^oC ICE → 0^oC WATER → 100^oC WATER → 100^oC STEAM

Q_2 ↓ mL_f ; Q_3 ↓ $ms_w\Delta T_2$; Q_4 ↓ mL_v

$Q = mL_f + ms_w\Delta T_2 + mL_v + Ms\Delta T \Rightarrow Q = m(L_f + s_w\Delta T_2 + L_v) + W\Delta T$

$Q = 5(80 + 1(100) + 540) + 20(100 - 0) \Rightarrow Q = 3600 + 2000 = 5600$ cal

Mixing of phases

When objects with different phases are mixed together

In this case temperature as well as phase can change.

Steps to solve numerical

Step - 1

First of all assume a reference temperature.

ice + water → T_{ref} = 0 °C water

ice + steam → T_{ref} = 100 °C water

water + steam → T_{ref} = 100 °C water

Step - 2

Calculate required & supplied heat corresponding to reference. If

$Q_R = Q_S$ → $T_{mix} = T_{reference}$

$Q_R > Q_S$ → $T_{mix} \leq T_{reference}$

$Q_R > Q_S$ → $T_{mix} \geq T_{reference}$

Geyser Problem

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$\text{Power(P)} = \frac{\text{Heat}}{\text{Time}} \qquad \left(\because \text{Heat} = Q = (ms\Delta T)\right)$$

$$\frac{P}{4.2} = \frac{(ms\Delta T)}{t} \qquad P = \frac{J}{\text{sec}} \text{ or watt} = \frac{J}{\text{sec}(4.2)}$$

$$\frac{P}{4.2} = \left(\frac{m}{t}\right) s\Delta T$$

$\left(\frac{m}{t}\right)$ = rate of flow of water in gm/s

s_{water} = 1 cal/gm°C

Illustration 27:

10g Ice at 0°C is mixed with 10 g water at 60°C. Find the final temperature of mixture.

Solution:

10g ice at 0°C → Q = mL = 800 Cal. → 10 g water, 0°C

10g water at 60°C → Q = msΔT = 10 × 60 × 1 = 600 cal. → 10 g water, 0°C

Ice receive 600 cal from water while it requires 800 cal.

so, ice remaining is

200 = mL

$\Rightarrow m = \frac{200}{L} = 2.5g$

and water is

= 10 + 7.5

= 17.5 g

Final temp = 0°C

water = 17.5 g, Ice = 2.5 g

Illustration 28:

Find the quantity of heat required to convert 40 g of ice at –20°C into water at 20°C. Given $L_f = 0.336 \times 10^6$ J/kg. Specific heat of ice = 2100 J/kg–K, specific heat of water = 4200 J/kg–K

Solution:

Heat required to raise the temperature of ice from –20°C to 0°C = 0.04 × 2100 × 20 = 1680 J

Heat required to convert the ice into water at 0°C = mL = 0.04 × 0.336 × 106 = 13440 J

Heat required to heat water from 0°C to 20°C = 0.04 × 4200 × 20 = 3360 J

Total heat required = 1680 + 13440 + 3360 = 18480 J

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Illustration 29:

Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensed? Latent heat of steam = 536 cal/g.

Solution:

Heat required by (calorimeter + water)

$Q = (m_1c_1 + m_2c_2)\,\Delta\theta = (0.02 + 1.1 \times 1)\,(80 - 15) = 72.8$ kcal

If m is mass of steam condensed, then heat given by steam

$Q = mL + mc\,\Delta\theta = m \times 536 + m \times 1 \times (100 - 80) = 556\,m \;\; \therefore \; 556\,m = 72.8$

$\therefore$ Mass of steam condensed $m = \dfrac{72.8}{556} = 0.130$ kg

Illustration 30:

An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground, it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take specific heat of iron 470 J/kg °C)

Solution:

$$mS\Delta\theta = \frac{3}{4}mgh \Rightarrow \Delta\theta = \frac{3\times10\times10}{4\times470} = 0.159°C$$

Illustration 31:

A bullet of mass 5 gm is moving with speed 400 m/s strike a target. Then calculate rise of temperature of bullet. Assuming all the loose in kinetic energy is converted into heat energy of bullet if its specific heat is 500J/kg°C.

Solution:

Loss in K.E = Heat energy

$$\frac{1}{2}mv^2 = ms\Delta\theta$$

$$\frac{400\times400}{2} = 500\times\Delta\theta \Rightarrow \Delta\theta = 160°C$$

Illustration 32:

1 KW geyser is used to raise the temperature of water coming out a rate of 100 $\dfrac{cc}{sec}$ in it. Initial temperature was 10°C. If efficiency of geyser is 80% then find final temperature of water?

Solution:

$$\frac{P.\Delta T}{4.2} = ms\Delta T$$

$$\Rightarrow \frac{80}{100}\frac{1\times10^2}{4.2} = \frac{m}{\Delta t}(1)(T_f - 10°) \Rightarrow \frac{80}{100}\times\frac{10^3}{4.2} = \frac{v.s.}{\Delta t}(T_f - 10°)$$

$$\Rightarrow \frac{80}{100}\times\frac{10^3}{4.2} = 100(1)(T_f - 10°) \Rightarrow T_f = 11.9°C$$

Change of States

- **Melting**
  Conversion of solid into liquid state at constant temperature is known as melting.
- **Boiling**
  Evaporation within the whole mass of the liquid is called boiling. Boiling takes place at a constant temperature known as boiling point. A liquid boil when the saturated vapour pressure on its surface is equal to atmospheric pressure. Boiling point reduces on decreasing pressure.

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- **Evaporation**
  Conversion of liquid into vapours at all temperatures is called evaporation. It is a surface phenomenon. Greater the temperature, faster is the evaporation. Smaller the boiling point of liquid, more rapid is the evaporation. Smaller the humidity, more is the evaporation. Evaporation increases on decreasing pressure that is why evaporation is faster in vacuum.
- **Heat of evaporation**
  Heat required to change unit mass of a liquid into vapour at a given temperature is called heat of evaporation at that temperature.
- **Sublimation**
  Direct conversion of solid into vapour state is called sublimation.
- **Heat of sublimation**
  Heat required to change unit mass of solid directly into vapours at a given temperature is called heat of sublimation at that temperature.
  Camphor and ammonium chloride sublimates on heating in normal conditions.
  A block of ice sublimates into vapours on the surface of moon because of very–very low pressure on its surface
- **Condensation**
  The process of conversion from gaseous or vapour state to liquid state is known as condensation.
  These materials again get converted to vapour or gaseous state on heating.
- **Hoar frost**
  Direct conversion of vapours into solid is called hoar frost. This process is just reverse of the process of sublimation.
  Ex. : Formation of snow by freezing of clouds.

Phase of a substance and phase diagram

The phase of a substance is defined as its form which is homogeneous, physically distinct and mechanically separable from the other forms of that substance.

Phase diagram

A phase diagram is a graph in which pressure (P) is represented along the y–axis and temperature (T) is represented along the x–axis.

Characteristics of Phase diagram

(i) Different phases of a substance can be shown on a phase diagram.

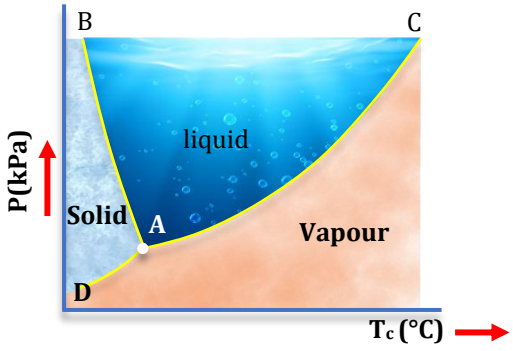
(ii) A region on the phase diagram represents a single phase of the substance, a curve represents equilibrium between two phases and a common point represents equilibrium between three phases.

(iii) A phase diagram helps to determine the condition under which the different phases are in equilibrium.

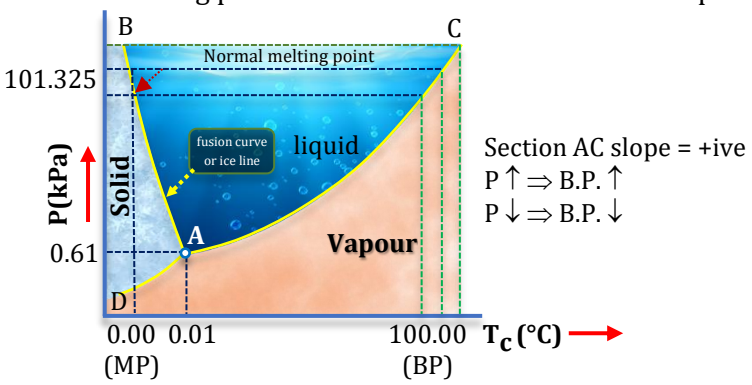
(iv) A phase diagram is useful for finding a convenient way in which a desired change of phase can be produced.

Phase Diagram for Water

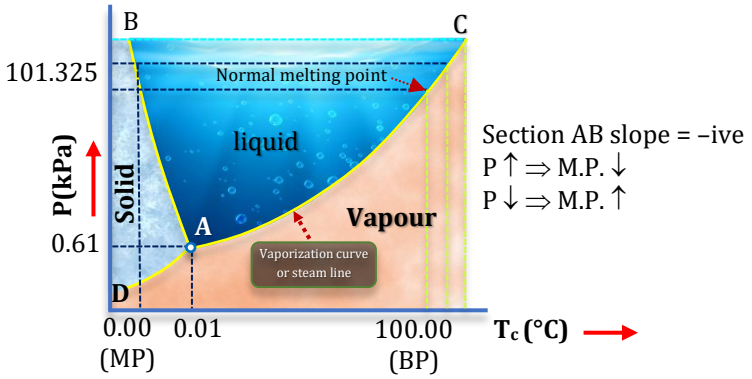
A phase diagram of water is a graph in which pressure (P) is represented along the y–axis and temperature (T) is represented along x-axis.



(i) Along curve AB, ice and water can remain in equilibrium. This curve is called fusion curve or ice line. This curve shows that the melting point of ice decreases with increase in pressure.

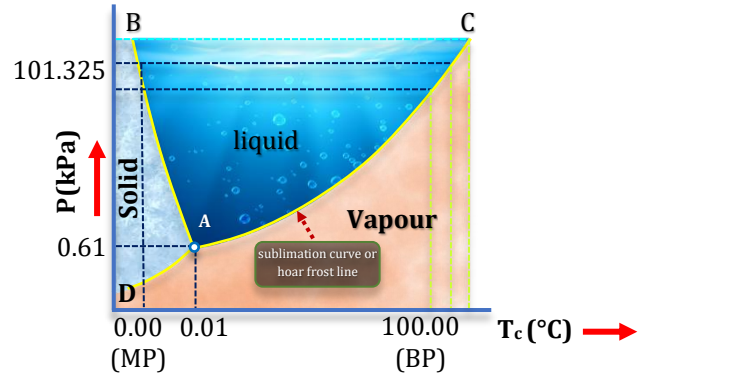


(ii) Along the curve AC, water and water vapor can remain in equilibrium. The curve is called vaporization curve or steam line. The curve shows that the boiling point of water increases with increase in pressure.



(iii) Along the curve AD, ice and water vapour can remain in equilibrium.

(iv) This curve is called sublimation curve or hoar frost line.



Note : Along any curve the two phases can coexist in equilibrium.

- At triple point of water
  Temperature = 0.01 °C ≃ 273.16 K
  Pressure = 4.6 mm of Hg
- Effect of pressure variation on melting point temperature and boiling point temperature can be analysed by phase indicator diagram.

Triple point of water and Regelation

The three curves in the phase diagram of water meet at a single point A, which is called the triple point of water. The triple point of water represents the coexistence of all the three phases of water ice water and water vapor in equilibrium. The pressure corresponding to triple point of water is 6.03×10^{-3} atmosphere or 4.58 mm of Hg and temperature corresponding to it is 273.16K.

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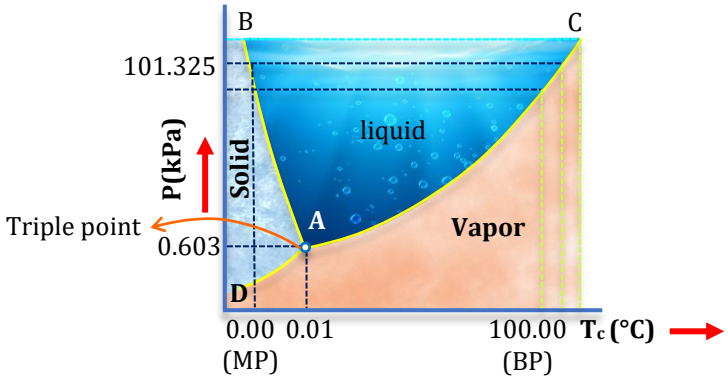


Significance of triple point of water

Triple point of water represents a unique condition, and it is used to define the absolute temperature. While making Kelvin's absolute scale, upper fixed point is 273.16 K and lower fixed point is 0 K. One kelvin of temperature is $\frac{1}{273.16}$ part of the temperature of triple point of water.

Example :

(1) A bottle is filled with water at 30°C on opening at moon then water will boil and vaporized.

(2) At higher altitudes of mountain, food cannot be cooked properly.

Regelation

Regelation is the melting of ice caused by pressure and its re-solidification when the pressure is removed. Ice shrinks when it melts, and if pressure is applied, deliberately promoting shrinkage, it is found that melting is thereby assisted. In other words, melting of cold ice is ordinarily effected by raising the temperature, but if pressure is present to help with the shrinkage the temperature need not be raised so much.

Ice heals up after being cut through by the wire. Melting takes place under the wire because pressure lowers the melting temperature. Refreezing (regelation) occurs above the wire when the water escapes to normal pressure again.

Increase of pressure lowers the melting (or freezing) point of water. Conversely, if a substance expands on melting, the melting point is raised by pressure.

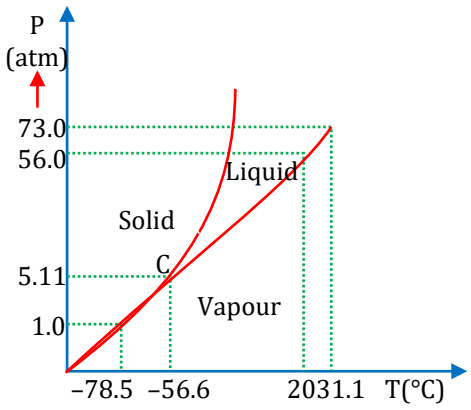
Illustration 33:

Answer the following questions based on the P-T phase diagram of carbon dioxide:

(i) At what temperature and pressure can the solid, liquid and vapour phases of CO_2 co-exist in equilibrium?

(ii) What is the effect of decrease of pressure on the fusion and boiling point of CO_2?

(iii) Is CO_2 solid, liquid or gas at (a) −70° C under 1 atm, (b) −60° C under 10 atm, (c) 15° C under 56 atm?



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Solution:

The P-T phase diagram for CO_2 is shown in the attached figure.

(i) The three phases can coexist at triple point. From the graph, it is at −56.6° C and 5.11 atm.

(ii) For carbon dioxide, the critical temperature is 31.1° C and critical pressure is 73.0 atm. If the temperature of carbon dioxide is more than 31.1° C, it cannot be liquefied, however large pressure we may apply.

(iii) Carbon dioxide will be (a) a vapour, at =70° C under 1 atm. (b) a solid, at −6° C under 10 atm (c) a liquid, at 15° C under 56 atm.

BEGINNER'S BOX-2

1. A bullet of mass 10 g is moving with speed 400m/s. Find its kinetic energy in calories?

2. Calculate amount of heat required to convert 1 kg steam from 100°C to 200°C steam?

3. Calculate heat required to raise the temperature of 1 g of water by 1°C?

4. 420 J of energy supplied to 10 g of water will raise its temperature by?

5. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3 . Find the ratio of their thermal capacities for unit volume?

6. Heat releases by 1 kg steam at 150°C if it is converted into 1 kg water at 50°C.

7. 200 g water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20°C. Find the heat supplied to the water.

8. A bullet of mass 5 gm is moving with speed 400 m/s strike a target. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is 500J/kg°C.

9. 1 kg ice at −10°C is mixed with 1 kg water at 100°C. Then find equilibrium temperature and mixture content.

10. 540 g of ice at 0°C is mixed with 540 g of water at 80°C. The final temperature of the mixture is (Given latent heat of fusion of ice = 80 cal/g and specific heat capacity of water = 1 cal/g°C)
(A) 0°C (B) 40°C (C) 80°C (D) less than 0°C

Modes of Heat Transfer

Heat is a form of energy which transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by any one of the following modes :

- **Conduction**
  The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.
- **Convection**
  The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant is fluids (liquids and gases).
- **Radiation**
  Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter–venning medium.

Conduction

It is a process of heat transfer in which heat is transferred from one particle to other particle without dislocation of particle from there equilibrium position is called conduction or thermal conduction.

- Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.
- Medium is necessary for conduction.
- Conduction is process which is possible in all states of matter.
- It's a slow process.
- In solids majorly, conduction takes place.

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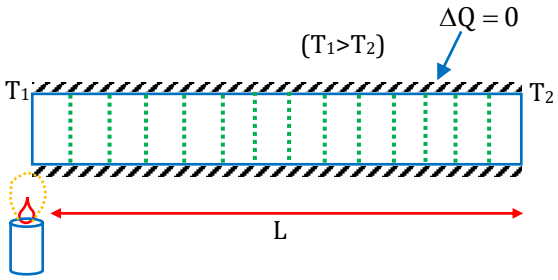
Let we heat a rod from its one end as shown in fig.

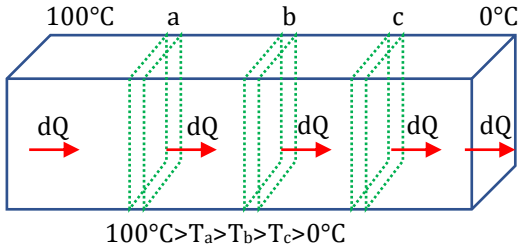


When we continue heat up its one end than slowly, other parts also become hot as time passes. firstly, all parts of rod absorb heat and after some time no heat is absorbed by any part. This is explained under two states :-

(1) **Variable state : -** In this state temperature of each and every part of the rod increases with time.

(2) **Transient State / Steady state :** After a long time, variable state comes when no heat is absorbed by any part. So, the temperature of every part is constant and decreases uniformly from hotter end to colder end.

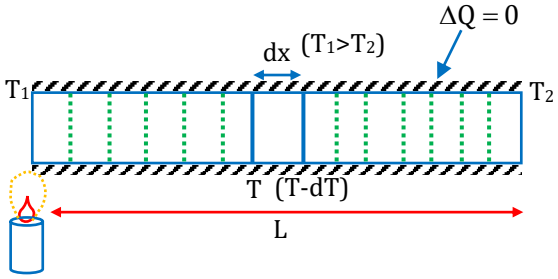
A rectangular, metallic conductor with its two faces at temperature 100 °C and 0 °C is shown in the figure. When the steady-state heat transfer is achieved, the temperatures of cross sections a, b, and c, become constant with time. The same amount of heat, dQ, flows through all such



Note : In steady state each point has different temperature but it remains constant.

Fourier's law of conduction

Rate of heat flow / Heat current / Thermal current (dQ/dt) :-



In given diagram, in steady state the heat flow through the rod is constant. The rate of heat flow $\left(\frac{dQ}{dt}\right)$ depends on: -

(1) Cross section Area (A) of rod

(2) Rate of change of temperature with distance $\left(-\frac{dT}{dx}\right)$ = Temperature gradient.

$$\therefore i_T = \frac{dQ}{dt} \propto A$$

$$\therefore i_T = \frac{dQ}{dt} \propto \left(-\frac{dT}{dx}\right)$$

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On Combining the relations,

$$i_T = \frac{dQ}{dt} \propto A\left(-\frac{dT}{dx}\right) \Rightarrow i_T = \frac{dQ}{dt} = -KA\left(\frac{dT}{dx}\right)$$

where K → coefficient of conductivity
if we take the whole length of rod then

Rate of heat flow $i_T = \frac{dQ}{dt} = \frac{KA}{L}(T_1 - T_2)$

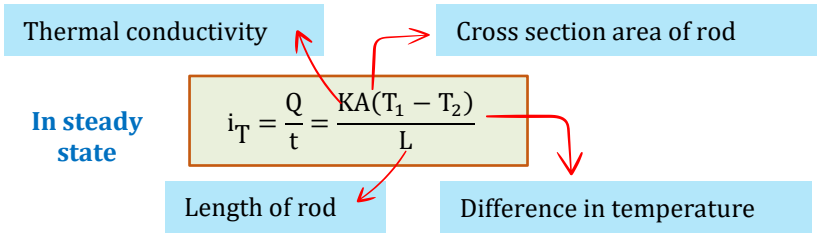

Illustration 34:

One face of an aluminium cube of edge 2 metre is maintained at 100 °C and the other end is maintained at 0 °C. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (thermal conductivity of aluminium is 209 W/m–°C)

Solution:

Heat will flow from the end at 100°C to the end at 0°C.

Area of cross-section perpendicular to direction of heat flow, A = 4m² then $\frac{Q}{t} = KA\frac{(T_H - T_C)}{L}$

$$Q = \frac{(209 W/m°C)(4m^2)(100°C - 0°C)(5 sec)}{2m} = 209 kJ$$

Illustration 35:

The ratio of the diameters of two metallic rods of the same material is 4 : 1 and their lengths are in the ratio 1 : 2. If the temperature difference between them are equal, the rate of flow of heat in them will be in the ratio of

Solution:

$$\frac{dQ}{dt} = H = \frac{KA(\Delta T)}{L}$$

$$H \propto \frac{A}{\ell} \propto \frac{r^2}{\ell}$$

$$\frac{H_1}{H_2} = \left[\frac{r_1}{r_2}\right]^2 \times \left[\frac{\ell_2}{\ell_1}\right] = \left[\frac{4}{1}\right]^2\left[\frac{2}{1}\right] = \frac{16\times 2}{1} = \frac{32}{1}$$

Illustration 36:

One end of a brass rod 2m long and having 1 cm radius is maintained at 250°C. When a steady state is reached, the rate of heat flow across any cross–section is 0.5 cal s^{-1}. What is the temperature of the other end K = 0.26 cal s^{-1} cm^{-1} $°C^{-1}$.

Solution:

$\frac{Q}{t}$ = 0.5 cal s^{-1} ; r = 1 cm

∴ Area A = πr^2 = 3.142 × 1 cm^2 = 3.142 cm^2

L = Length of rod = 2m = 200 cm, T_1 = 250°C, T_2 = ?

We know $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$ or $(T_1 - T_2) = \frac{Q}{t} \times \frac{\Delta x}{kA} \times \frac{0.5 \times 200}{0.26 C^{-1} \times 3.142} = 122.4°C$

∴ T_2 = 250°C – 122.4°C = 127.6°C

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Thermal Conductivity and Thermal Resistance

Thermal conductivity (K)

- It is the measure of the ability of a substance to conduct heat through it.
- Substance that are good thermal conductors have large thermal conductivity values, whereas good thermal insulators have low thermal conductivity values.
- It depends on nature of material.

  Order of thermal conductivity $\boxed{Ag > Cu > Au > Al}$
- K for Ag maximum is (410 W/mk) and For Freon - 12 minimum is (0.008 W/mK)
- For an ideal or perfect conductor of heat the value of $K = \infty$
- For an ideal or perfect bad conductor or insulator the value of K = 0
- Unit : SI $\rightarrow \frac{W}{mK}$ or $J\,s^{-1}\,m^{-1}\,K^{-1}$; CGS $\rightarrow$ Cal/cm – sec–°C
- Dimensions : $M^1\,L^1\,T^{-3}\,\theta^{-1}$

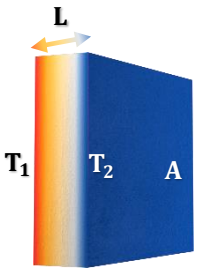
Application of Thermal Conduction

- In winter, the iron chairs appear to be colder than the wooden chairs.
- For cooking the food, low specific heat and high conductivity utensils are most suitable.
- Cooking utensils are made of aluminium and brass whereas their handles are made of wood/Bakelite.
- Ice is covered in gunny bags to prevent melting of ice.
- We feel warm in woollen clothes and fur coat.
- Two thin blankets are warmer than a single blanket of double the thickness.
- Birds often swell their feathers in winter.
- A new quilt is warmer than old one.

Thermal Resistance to Conduction

It is the obstruction offered to the flow of heat through the conductor when we compare the flow of heat to the flow of electricity.

It is defined as the ratio of temperature difference to the heat current (rate of flow of heat)

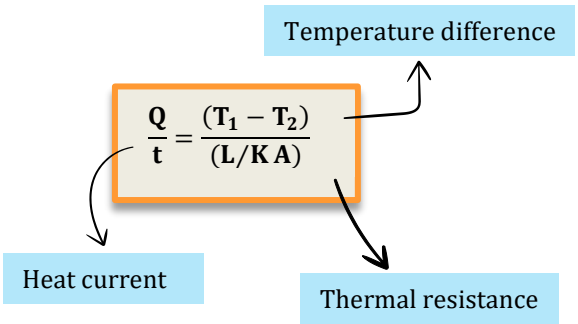


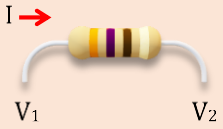
For a slab of cross-section A, Lateral thickness L and thermal conductivity K

$$R_T = \frac{T_1 - T_2}{i_T} = \frac{T_1 - T_2}{KA(T_1 - T_2)/L}$$

$$R_T = \frac{L}{KA}$$

Electrical Analogy for Thermal Conduction



Electric Conduction	Thermal Conduction
	
Electric current $I=\frac{dq}{dt}$	Heat current $H=\frac{dQ}{dt}$
$I=\frac{V_1-V_2}{R}$ R is Electrical resistance	$H=\frac{T_1-T_2}{R_T}$ R_T is Thermal resistance
$R=\frac{\rho L}{A}=\frac{L}{\sigma A}$	$R_T=\frac{L}{KA}$

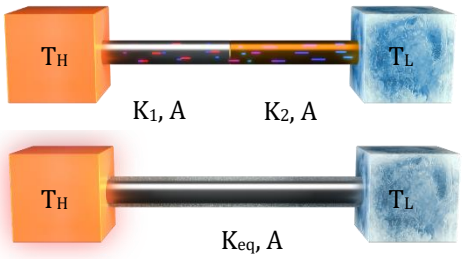
Equivalent Thermal Conductivity in series Combination

In series combination

$$R_{eq}=R_1+R_2$$

$$\Rightarrow \frac{(L_1+L_2)}{K_{eq}.A}=\frac{L_1}{K_1A}+\frac{L_2}{K_2A}$$

Rods joined end to end



Equivalent thermal conductivity is

$$K_{eq}=\frac{L_1+L_2}{\frac{L_1+L_2}{K_1+K_2}}=\frac{\Sigma L_i}{\Sigma\frac{L_i}{K_i}}$$

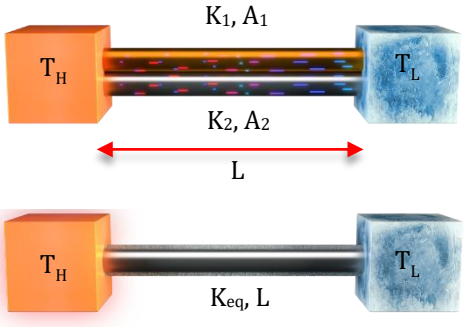
- If lengths of rod is same then ($L_1 = L_2 = L$)

$$K_{eq}=\frac{2K_1K_1}{K_1+K_2}$$

Equivalent Thermal Conductivity in Parallel Combination

In parallel combination

Rods are joined side by side



$$\Rightarrow \quad \frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$$

$$\Rightarrow \quad \frac{K_{eq.}(A_1+A_2)}{L}=\frac{K_1A_1}{L}+\frac{K_2A_2}{L}$$

Equivalent thermal conductivity is

$$\Rightarrow \quad K_{eq}=\frac{K_1A_1+K_2A_2}{A_1+A_2}=\frac{\Sigma K_iA_i}{\Sigma A_i}$$

- If Area of both rods are same then ($A_1 = A_2 = A$)

$$K_{eq}=\frac{K_1+K_2}{2}$$

$$(K_1<K_{eq}<K_2)$$

Temperature of junction

When we have to calculate the temperature of junction then we equate the rate of heat flow from both parts.

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$$

$$\Rightarrow \quad \frac{K_1 A}{L_1}(T_1 - T_0) = \frac{K_2 A}{L_2}(T_0 - T_2)$$

$$T_0 = \frac{K_1 L_2 T_1 + K_2 L_1 T_2}{K_1 L_2 + K_2 L_1}$$

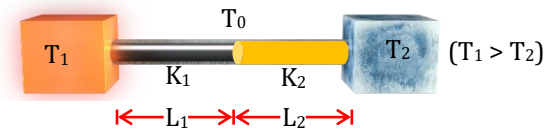


Illustration 37:

The temperature gradient in a rod of 0.5 m length is 80°C/m. It the temperature of hotter end of the rod is 30°C, then the temperature of the cooler end is :-

Solution:

$$\frac{\Delta T}{\Delta x} = 80^\circ C/m \Rightarrow \frac{30 - T}{0.5} = 80 \Rightarrow T = -10^\circ C$$

Illustration 38:

Find equivalent thermal conductivity in the given figure.

Solution:

Area = A

$$K_{eq} = \frac{\ell_1 + \ell_2}{\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2}}$$

$$K_{eq} = \frac{3L}{\frac{L}{K} + \frac{2L}{3K}} \Rightarrow \frac{9K}{5}$$

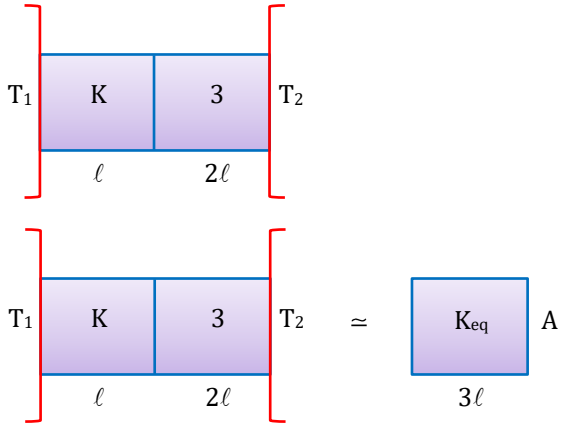


Illustration 39:

Find the equivalent conductivity (K_{eq})

Solution:

$$K_{eq} = \frac{L_{eq}}{\frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + \frac{\ell_3}{K_3}} = \frac{8\ell}{\frac{\ell}{K} + \frac{4\ell}{2K} + \frac{3\ell}{K}} \Rightarrow \frac{16K}{2+4+6} = \frac{4}{3}K$$

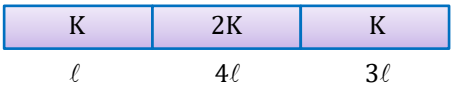


Illustration 40:

Find equivalent thermal conductivity (K_{eq})

Solution:

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2} = \frac{2K(3A) + 2K(A)}{4A} = \frac{8KA}{4A} = 2K$$

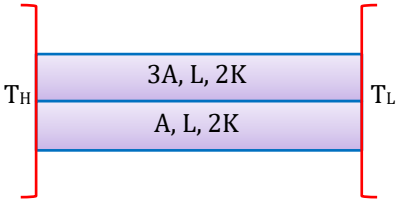


Illustration 41:

Find temperature T_A (temp. at point A) for the given figure.

Solution:

$$\frac{KA}{2}(100 - T_0) = \frac{KA}{3}(T_0 - 20)$$

$$\Rightarrow 150 - \frac{3}{2}T_0 = T_0 - 20$$

$$\Rightarrow T_0 = 68\ ^\circ C$$

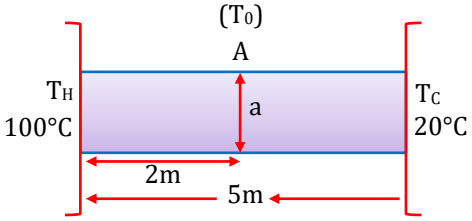
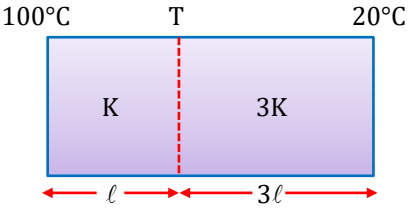


Illustration 42:

Find the temperature of junction for following figure.



Area is same.

Solution:

$i_1 = i_2$

$$\left(\frac{Q}{T}\right)_1 = \left(\frac{Q}{T}\right)_2$$

$$\therefore \frac{Q}{t} = \frac{KA}{\ell}(T_1 - T_2)$$

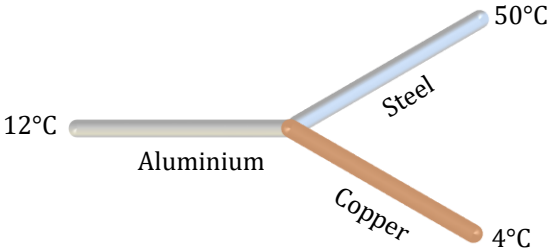
$$\left(\frac{KA\Delta T}{\ell}\right)_1 = \left(\frac{KA\Delta T}{\ell}\right)_2 \Rightarrow K\frac{A(100-T)}{\ell} = 3K\frac{A(T-20)}{3\ell}$$

$\Rightarrow 100 - T = T - 20 \Rightarrow 2T = 120 \Rightarrow T = 60°C$

Illustration 43:

Three identical rods of length 1m each, having cross-section area of $1cm^2$ each and made of Aluminium, Copper and Steel respectively are maintained at temperatures of 12°C, 4°C and 50°C respectively at their separate ends. Find the temperature of their common junction.

[K_{Cu} = 400 W/m-K, K_{Al} = 200 W/m-K, K_{steel} = 50 W/m-K]



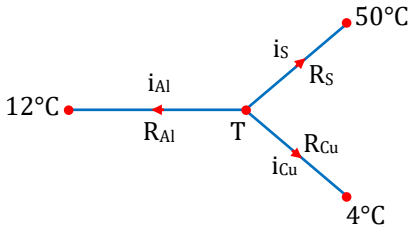
Solution:

$$R_{Al} = \frac{L}{KA} = \frac{1}{200\times10^{-4}} = \frac{10^4}{200}$$

Similarly, $R_{steel} = \frac{10^4}{50}$ and $R_{copper} = \frac{10^4}{400}$

Let temperature of common junction = T

then from Kirchhoff's current law, $i_{Al} + i_{steel} + i_{Cu} = 0$



$$\Rightarrow \frac{T-12}{R_{Al}} + \frac{T-50}{R_{steel}} + \frac{T-4}{R_{Cu}} = 0$$

$\Rightarrow (T - 12)\,200 + (T - 50)\,50 + (T - 4)\,400 = 0$

$\Rightarrow 4(T - 12) + (T - 50) + 8\,(T - 4) = 0$

$\Rightarrow 13T = 48 + 50 + 32 = 130$

$\Rightarrow T = 10°C$

Illustration 44:

Two vessels of different materials are identical in size and wall–thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

Solution:

Let K_1 and K_2 be the coefficients of thermal conductivity of the materials, and t_1 and t_2 be the time in which ice melts in the two vessels. Since both the vessels are identical, so A and L in both the cases is same.

Now

$$Q = \frac{K_1A(T_1 - T_2)t_1}{L} = \frac{K_2A(T_1 - T_2)t_2}{L}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25\text{ min}}{10\text{ min}} = \frac{5}{2}$$

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Illustration 45:

Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is –20°C and that of the external surface of the second plate is 20°C. What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.

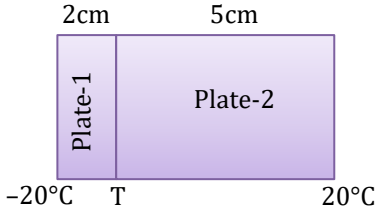
Solution:

Rate of flow of heat in the plates is $\frac{Q}{t}=\frac{K_1A(T_1-T)}{L_1}=\frac{K_2A(T-T_2)}{L_2}$ ...(i)

(i) Here $T_1 = -20°C$, $T_2 = 20°C$,

$L_1 = 2$ cm $= 0.02$ m, $L_2 = 5$ cm $= 0.05$ m and $K_1 = K_2 = K$

$\therefore$ Equation (i) becomes $\frac{KA(-20-T)}{0.02}=\frac{KA(T-20)}{0.05}$



$\therefore 5(-20-T) = 2(T-20)$

$\Rightarrow -100 - 5T = 2T - 40$

$\Rightarrow 7T = -60 \quad \Rightarrow T = -8.6°C$

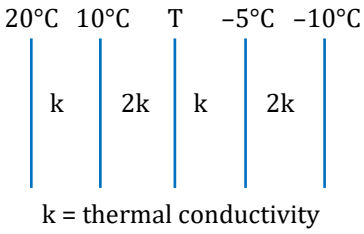
(ii) $\frac{K_1}{K_2}=\frac{2}{5}$ or $K_1=\frac{2}{5}K_2$

$\therefore$ from equation (i) $\frac{2/5K_2A(-20-T)}{0.02}=\frac{K_2A(T-20)}{0.05}$

$\Rightarrow -20 - T = T - 20 \quad \therefore T = 0°C$

Illustration 46:

The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.



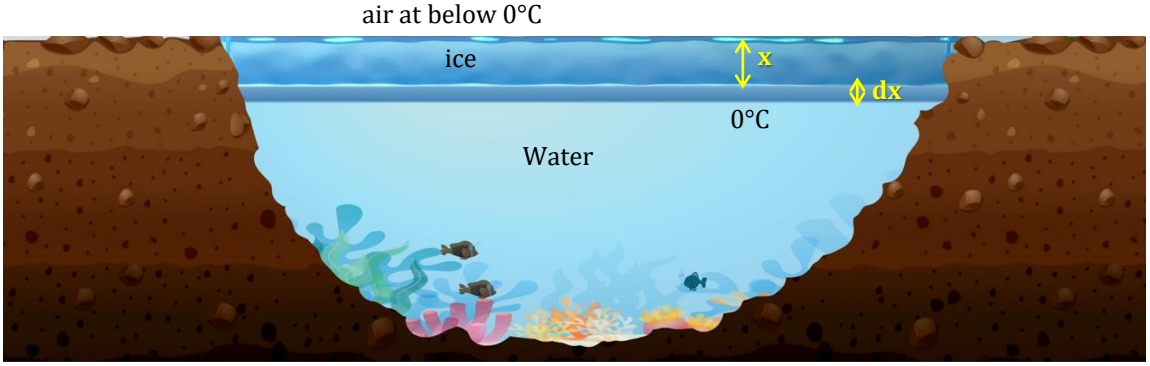
k = thermal conductivity

Solution:

$\frac{\Delta Q}{\Delta t}$ = same

$\frac{KA(20-10)}{\ell}=\frac{2KA(10-T)}{\ell} \Rightarrow 2T = 10 \Rightarrow T = 5°C$

Growth of Ice on Lake



In winter atmospheric temperature falls below 0°C and water in the lake start freezing. Let at time t thickness of ice on the surface of the lake = x and air temperature = –T° C

The temperature of water in contact with the lower surface of ice = 0°C

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Let area of the lake = A

Heat escaping through ice in time dt is $dQ = KA\frac{[0-(-T)]}{x}dt$

Due to escape of this heat increasing extra thickness of ice = dx

Mass of this extra thickness of ice is m = $\rho V = \rho$ A.dx

$dQ = mL = (\rho\ A.dx)L$

$\therefore \quad KA\frac{T}{x}dt = (\rho\ A.dx)L \Rightarrow dt = \frac{\rho L}{KT}x\,dx$

So, time taken by ice to grow a thickness x is $t = \frac{\rho L}{KT}\int_0^x x\,dx = \frac{1}{2}\frac{\rho L}{KT}x^2$

So, time taken by ice to grow from thickness x_1 to thickness x_2 is

$t = t_2 - t_1 = \frac{1}{2}\frac{\rho L}{KT}(x_2^2 - x_1^2)$ and $\boxed{t \propto (x_2^2 - x_1^2)}$

- Time taken to double and triple the thickness ratio
  $t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2$
  $t_1 : t_2 : t_3 :: 1 : 4 : 9$
- Ratio of time taken to form thickness
  $(0 \to x) : (x \to 2x) : (2x \to 3x)$
  $\Delta t_1 : \Delta t_2 : \Delta t_3 :: (x^2 - 0^2) : (2x^2 - x^2) : (3x^2 - 2x^2)$
  $\Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$

Illustration 47:

Ice starts freezing in a lake with water at 0°C when the atmospheric temperature is −10°C. If the time taken for 1cm of ice to be formed is 12min, the time taken for the thickness of the ice to change from 1 to 2cm will be

Solution:

Time taken by ice to grow a thickness of

$t = \frac{eL}{2K\theta}y^2$

Hence time interval to change thickness from 0 to y, from y to 2y and so on will be in the ratio.

$\Delta t_1 : \Delta t_2 : \Delta t_3 = (1 - 0^2) : (2^2 - 1^2) : (3^2 - 2^2)$

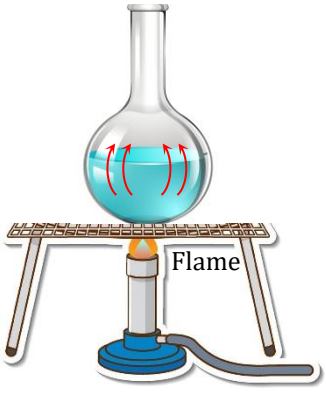
$\Delta t_1 : \Delta t_2 : \Delta t_3 = 1 : 3 : 5$

According to question

$\Delta t_2 = 3\Delta t_1 = 3 \times 12 = 36$ min.

Convection

Convection requires a medium and is the process in which heat is transferred from one place to other by actual movement of heated substance (usually fluid). The type of convection which results from difference in densities is called natural convection (for example, a fluid in a container heated through its bottom). However, if a heated fluid is forced to move by a blower, fan or pump, the process is called forced convection.



Phenomena Based on convection:

(i) Land and sea breezes:

The heat from the Sun is absorbed more rapidly by land than by sea–water. Moreover, the specific heat of land is low as compared to that of sea–water. Consequently, the rise in temperature of land is higher as compared to that of sea–water. To sum–up, land is hotter than the sea during day time. As a result of this, the colder air over the sea blows towards the land. This is called sea–breeze. At night, air blows from land towards sea. This is called land breeze.

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(ii) Formation of trade winds:

The surface of Earth near the equator gets heated strongly. So, the air in contact with the surface of Earth at the expands and rises upwards. As a result of this, a low pressure is created at the equator. At the poles, the air in the upper atmosphere gets cooled and comes down. So, a high pressure is created at the poles. Due to difference of pressures at the poles and equator, the air at the poles moves towards the equator, rises up, moves towards poles and so on. In this way, a wind is formed in the atmosphere.

The rotation of the Earth also affects the motion of the wind. Due to anti–clockwise rotation of Earth the warm wind blowing from equator to north drifts towards east. The steady wind blowing from north–East to equator, near the surface of Earth, is called trade wind.

(iii) Monsoons:

In summer, the peninsular mass of central Asia becomes more strongly heated than the water of the Indian Ocean. This is due to the fact that the specific heat of water is much higher than that of the soil and rocks. Hot air from the heated land mass rises up and moves towards the Indian ocean. Air filled with moisture flows over the Indian ocean on the south towards heated land mass. When obstructed by mountains, the moist air rushes upwards to great height. In the process, it gets cooled. Consequently, the moisture condenses and falls as rain.

(iv) Ventilation:

Ventilator of exhaust fan in a room help of remove impure and warm air from a room. The fresh air from outside blows into the room. This is all due to the forced convection current set up in the room.

(v) To regulate temperature in the human body:

Heat transfer in the human body involves a combination of mechanisms. These together maintain a remarkably uniform temperature in the human body inspite of large changes in environmental conditions. The chief internal mechanism is forced convection. The heart serves as the pump and the blood as the circulating fluid.

Some important points:

- Natural convection takes place from bottom to top due to gravity while forced convection in any direction.
- In case of natural convection, convection currents move warm air upwards and cool air downwards. This is why heating is done from base, while cooling from the top.
- Natural convection is not possible in a gravity free region such as a freely falling lift or an orbiting satellite.
- Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.
- The forced convection of blood in our body by a pump (heart) helps in keeping the temperature of body constant.
- Convection takes place in fluids (gases and liquid)

Key Points

- For heat propagation via natural convection, temperature gradient exists in vertical direction and not in horizontal direction.
- Most of heat transfer that is taking place on Earth is by convection, the contribution due to conduction and radiation is very small.

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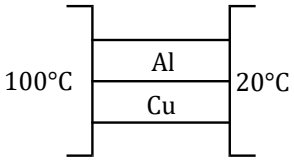
BEGINNER'S BOX-3

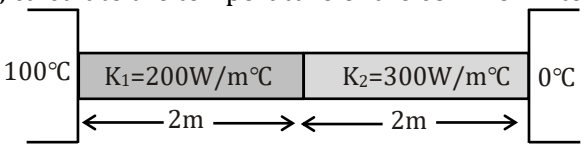
1. Explain why :
(a) a brass tumbler feels much colder than a wooden tray on a chilly day
(b) two layers of cloth of equal thickness provide warmer covering than a single layer of cloth of double thickness?
(c) mud-houses are colder in summer and warmer in winter?
(d) In winter birds sit with their wings spread out?
(e) Woollen clothes are warmer than cotton clothes?

2. Two metal cubes with 3 cm-edges of copper and aluminium are arranged as shown in figure. Find

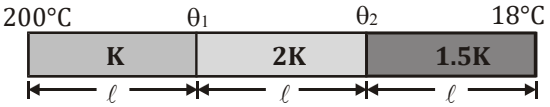


(a) The total thermal current from one reservoir to the other.
(b) The ratio of the thermal current carried by the copper cube to that carried by the aluminium cube. Thermal conductivity of copper is 60 W/m–K and that of aluminium is 40 W/m–K.

3. For shown situation, calculate the temperature of the common interface.

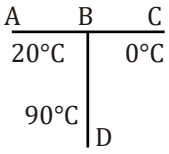


4. Calculate θ_1 and θ_2 in shown situation.



5. The temperature at the ends of a uniform rod of length 100 cm are respectively 95°C and 5°C. What will be the temperature at a point 30 cm far from the hotter end ? Also calculate the temperature gradient.

6. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20°C, 90°C and 0°C. Find the ratio of length BD and BC if there is no heat flow in AB.



Radiation and Prevost's Theory

Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. When a body is heated and placed in vacuum, it loses heat even when there is no medium surrounding it. The heat cannot go out from the body by the process of conduction or convection since both of these processes require the presence of a material medium between source and surrounding objects. The process by which heat is lost in this case is called radiation. This does not require the presence of any material medium.

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It is radiation by which heat from the Sun reaches the Earth. Radiation has the following properties:

(a) Radiant energy travels in straight lines and when some object is placed in the path, its shadow is formed at the detector.

(b) It is reflected and refracted or can be made to interfere. The reflection or refraction are exactly as in case of light.

(c) It can travel through vacuum.

(d) Intensity of radiation follows the law of inverse square.

(e) Thermal radiation can be polarised in the same way as light by transmission through a Nicol prism.

(f) Radiation takes place in solid, liquid & gases.

All these and many other properties establish that heat radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency. The wavelength of thermal radiation is larger than that of visible light.

- When radiation passes through any medium then radiations slightly absorbed by medium according to its absorptive power so temperature of medium slightly increases.
- In order to obtain a spectrum of radiation, a special prism is used like KCl prism, Rock salt prism Fluorspar prism. Normal glass prism or Quartz prism cannot be used (because it absorbs some radiation).
- Radiation intensity is measured with a specific device named as Bolometer.
- Heat radiation are always obtained in infra–red region of electromagnetic wave spectrum so they are called Infra-red rays.
- Thermal radiation when incident on a surface, then exert pressure on the surface which is known as Radiation Pressure.

Note: All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of radiations

Basic Fundamental definitions

- **Absorptive power or absorptive coefficient (a) :** The ratio of amount of radiation absorbed by a surface (Qa) to the amount of radiation incident (Q) upon it, is defined as the coefficient of absorption $a = \frac{Q_a}{Q}$.

  It is unitless and dimensionless.

- **Spectral absorptive power** (a_λ) $a_\lambda = \frac{Q_{a_\lambda}}{Q_\lambda}$: Also called monochromatic absorptive coefficient

  At a given wavelength $a = \int_0^\infty a_\lambda d\lambda$. For ideal black body a_λ and $a = 1$, a and a_λ are unitless

- **Emissive power (e):** The amount of heat radiation emitted by unit surface area in unit second at a particular temperature.
  UNIT: SI $\rightarrow$ J/m²–s or watt/m²

Emissivity (e)

- **Absolute emissivity or emissivity:** Radiation energy given out by a unit surface area of a body in unit time corresponding to unit temperature difference w.r.t. the surroundings is called Emissivity.
  Unit: SI $\rightarrow$ W/m² K

- **Relative emissivity (e_r) :** $e_r = \frac{Q_{GB}}{Q_{IBB}} = \frac{e_{GB}}{E_{IBB}} = \frac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

  GB = gray or general body, IBB = Ideal black body

  (i) No unit (ii) For ideal black body $e_r = 1$ (iii) range $0 < e_r < 1$

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Prevost's Theory

According to Prevost's Theory, at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time.

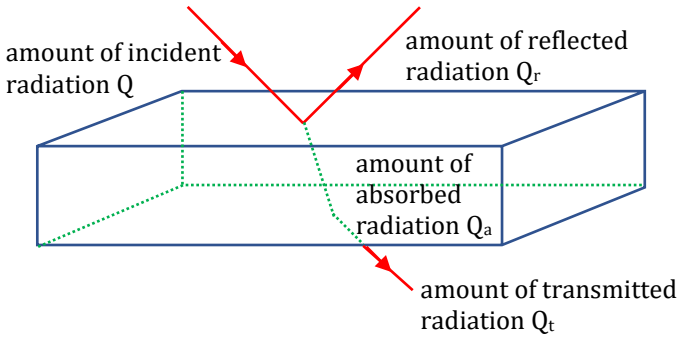
The relation between temperature difference of body with its surrounding decides whether the body experience cooling effect or heating effect.

- **When a cold body is placed in the hot surrounding:** The body radiates less energy and absorbs more energy from the surrounding, therefore the temperature of body increases. Here the body will experience heating effect.
  $T_B < T_S$
  $Q_{emission} < Q_{absorption}$
- **When a hot body placed in cooler surrounding:** The body radiates more energy and absorbs less energy from the surroundings. Therefore, temperature of body decreases. Here the body will experience cooling effect.
  $T_B > T_S$
  $Q_{emission} > Q_{absorption}$
- **When the temperature of a body is equal to the temperature of the surrounding:** The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body, therefore its temperature remains constant and the body is in thermal equilibrium with surrounding. Hence no heating and cooling effects are seen.
  $T_B = T_S$
  $Q_{emission} = Q_{absorption}$

Spectral, emissive, absorptive and transmittive power of a given body surface

Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts. (a) Reflection (b) Absorption (c) Transmission

From energy conservation


amount of incident radiation Q
amount of reflected radiation Q_r
amount of absorbed radiation Q_a
amount of transmitted radiation Q_t

$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1 \Rightarrow r + a + t = 1$$

Reflective Coefficient (r) = $\frac{Q_r}{Q}$

Absorptive Coefficient (a) = $\frac{Q_a}{Q}$

Transmittive Coefficient (t) = $\frac{Q_t}{Q}$

r = 1 and	a = 0,	t = 0	$\Rightarrow$	Perfect reflector
a = 1 and	r = 0,	t = 0	$\Rightarrow$	Ideal absorber (ideal black body)
t = 1 and	a = 0,	r = 0	$\Rightarrow$	Perfect transmitter (diathermanous)

Reflection power (r) = $\left[\frac{Q_r}{Q}\times 100\right]\%$

Absorption power (a) = $\left[\frac{Q_a}{Q}\times 100\right]\%$

Transmission power (t) = $\left[\frac{Q_t}{Q}\times 100\right]\%$

Note : For General body or Grey body (GB) ; $0 < r/a/t < 1$

★ Golden Key Points ★

- At absolute zero temperature (zero kelvin) all atoms of a given substance remains in ground state, so, at this temperature emission and absorption of radiation from any substance is impossible, so Prevost's heat energy exchange theory does not apply at this temperature, so it is called limiting temperature of Prevost's theory.
- With the help of Prevost's theory rate of cooling of any body w.r.t. its surroundings can be worked out (applied to Stefan Boltzmann law, Newton's law of cooling.)

Illustration 48:

Total radiation incident on body is 400 J, If 20% of incident radiation reflected back and 120 J is absorbed by body. Then find out transmittive power in percentage.

Solution:

$Q = Q_t + Q_r + Q_a \Rightarrow 400 = 80 + 120 + Q_t \Rightarrow Q_t = 200$

So transmittive power is $\frac{Q_t}{Q}\times 100\% = 50\%$

Ideal Black Body and Kirchhoff's Law

Basic Fundamental definitions

- Absorptive power or absorptive coefficient (a) : The ratio of amount of radiation absorbed by a surface (Qa) to the amount of radiation incident (Q) upon it, is defined as the coefficient of absorption $a = \frac{Q_a}{Q}$. It is unitless and dimensionless.
- **Spectral absorptive power** (a_λ) $a_\lambda = \frac{Q_{a_\lambda}}{Q_\lambda}$: Also called monochromatic absorptive coefficient

  At a given wavelength $a = \int_0^\infty a_\lambda d\lambda$. For ideal black body a_λ and a = 1, a and a_λ are unitless

Ideal Black Body

A body which absorbs all thermal radiations falling at its surface at low temperature and emits out all these absorbed radiations at high temperature, irrespective of their wave length is known as an ideal black body.

Characteristics of an Ideal Black Body

- The nature of emitted radiations from surface of ideal black body depends only on its temperature.
- The radiations emitted from surface of ideal black body are called as either full or white radiations.
- At any temperature the spectral energy distribution curve for surface of an ideal black body is always continuous and according to this concept if the spectrum of a heat source obtained is continuous then it must be ideal black body like kerosene lamp; oil lamp, heating filament etc.
- For an ideal black body
  $a = a_\lambda = 1$
  $r = t = 0$
  $e_r = 1$

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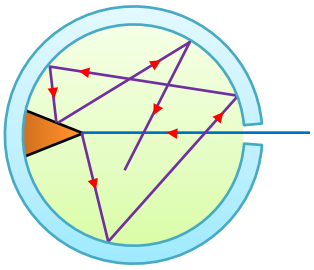
- At low temperature, surface of ideal black body is a perfect absorber and at a high temperature it proves to be a perfect emitter.

  Example of ideal black body

  There are two experimentally ideal black body

  (a) Wien's ideal black body

  (b) Ferry's ideal black body



Ferry's ideal black body

Note: An ideal black body need not be of black color (eg. Sun).

Kirchhoff's Law

It states that at a given temperature, for all bodies, the ratio of their spectral emissive power (e_λ) to spectral absorptive power (a_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at that temperature.

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant} \quad \Rightarrow \quad \left[\frac{e_\lambda}{a_\lambda}\right]_1 = \left[\frac{e_\lambda}{a_\lambda}\right]_2 = \text{constant}$$

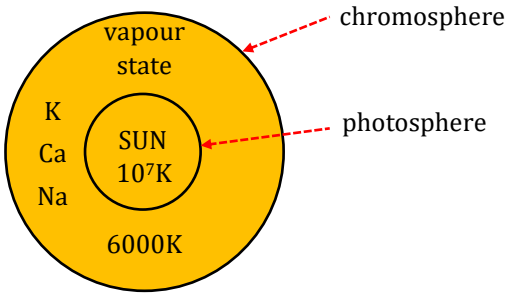
Hence, $e_\lambda \propto a_\lambda$

Thus, we can say, good absorbers are good emitters and bad absorbers are bad emitters.

Applications of Kirchhoff's Law

- **Fraunhofer's lines**

  Fraunhofer lines are dark lines in the spectrum of the Sun. When white light emitted from the central core of the Sun (Photosphere) passes through its atmosphere (chromosphere) some of the radiations are absorbed by the gases present, resulting in dark lines in the spectrum of Sun. At the time of total solar eclipse direct light rays emitted from photosphere cannot reach on the Earth and only rays from chromosphere are able to reach on the Earth surface. At that time we observe bright Fraunhofer lines.



- **In deserts days are hot and nights cold**

  Sand is rough and black, so it is a good absorber and hence in deserts, days (when radiation from Sun is incident on sand) will be very hot. Now in accordance with Kirchhoff's Law, a good absorber is a good emitter.

  So, nights (when send emits radiation) will be cold.

Golden Key Points

- For a constant temperature the spectral emissive power of an ideal black body is a constant parameter
- The practical confirmation of Kirchhoff's law carried out by Rishi apparatus and the main base of this apparatus is a Lessilie container.
- The main conclusion predicted from Kirchhoff's law can be expressed as

Good absorber	$\rightleftharpoons$	Good emitter
Bad absorber (at Low temperature)	$\rightleftharpoons$	Bad emitter (at high temperature)

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Illustration 49:

Which of the following is the example of ideal black body?

Black board, Kajal, A pin hole box.

Solution:

A body is said to be a black body if it perfectly absorbs and radiates the energy but does not reflect it. A pin hole box is an ideal black body. Black board and kajal although black in colour need not to be black body.

Illustration 50:

At a particular temperature and wavelength, the spectral emissive power and monochromatic absorptive power of a body are 10 and 8 units respectively. The emissive power of a black body at the same temperature will be

Solution:

According to Kirchhoff's law.

$\frac{e_\lambda}{a_\lambda} = E_\lambda$ here, given that $e_\lambda = 10$ and $a_\lambda = 8$

So, $E_\lambda = \frac{10}{8} = 1.25$

Stefan's Law

The amount of radiation emitted per second per unit area (E) by ideal black body is directly proportional to the fourth power of its absolute temperature (T).

$$E \propto T^4 \Rightarrow E = \sigma T^4$$

Where, $\sigma \rightarrow$ Stefan's constant = 5.67×10^{-8} watt /m^2 K^4 (universal constant)

Total radiation energy emitted by body having surface area A in time t :

(i) Ideal black body (IBB) : $Q_{IBB} = \sigma A T^4 t$

(ii) For any general body (GB) : $Q_{GB} = e_r \sigma A T^4 t$

SI Unit : Watt/m^2

[σ] : $M^1 L^0 T^{-3} \theta^{-4}$

Rate of emission of radiation

Let temperature of surrounding (T_0) < temperature of body (T)

Rate of emission of radiation from per unit area of ideal black body surface $E_1 = \sigma T^4$

Rate of absorption of radiation per unit area by ideal black body from surrounding $E_2 = \sigma T_0^4$

Net rate of loss of radiation per unit area from ideal black body surface is

$$E = E_1 - E_2 = \sigma T^4 - \sigma T_0^4 = \sigma(T^4 - T_0^4)$$

Net loss of radiation energy from entire surface area in time t

For ideal black body : $Q_{IBB} = \sigma A(T^4 - T_0^4)t$

For any general body : $Q_{GB} = eA\sigma(T^4 - T_0^4)t$

If in time 'dt' the net heat energy loss for ideal black body is 'dQ' and because of this its temperature falls by 'dT'

(I) Rate of heat loss :

Rate of heat loss is equal to the net rate at which a body is losing heat in terms of thermal radiations. It is also equal to emitted power or radiation emitted per second.

For ideal black body : $R_H = \frac{dQ}{dt} = \sigma A(T^4 - T_0^4)$

For any general body : $R_H = \frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$

Rate of heat loss depends on:

(a) Nature of radiating surface : Greater the emissivity (e), faster will be heat loss.

(b) Area of radiating surface : Greater the area of radiating surface, faster will be the heat loss.

(c) Temperature of radiating body :

Greater the temperature of radiating body, faster will be the heat loss.

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(II) Rate of fall in temperature (Rate of cooling) :

Rate of cooling is equal to the rate at which a temperature of a body falls.

$\because \frac{dQ}{dt} = ms\frac{dT}{dt}$

$\because \frac{dT}{dt} = \frac{1}{ms}\frac{dQ}{dt}$

For ideal black body : $R_F = \frac{dT}{dt} = \frac{\sigma A}{ms}(T^4 - T_0^4)$

For any general body : $R_F = \frac{dT}{dt} = \frac{e\sigma A}{ms}(T^4 - T_0^4)$

Rate of cooling depends on:

(a) Nature of radiating surface :
Greater the emissivity (e), faster will be the cooling.

(b) Area of radiating surface :
Greater the area of radiating surface, faster will be the cooling.

(c) Mass of radiating body :
Greater the mass of radiating body, slower will be the cooling.

(d) Specific heat of radiating body :
Greater the specific heat of radiating body, slower will be the cooling.

(e) Temperature of radiating body :
Greater the temperature of radiating body, faster will be the cooling.

Note: Rate of heat loss is independent of mass and specific heat of radiating body.

★ Golden Key Points ★

- If all of T, T_0, m, s, V, ρ, are same for different shape body then R_F and R_H will be maximum for the flat surface.
- If a solid and hollow sphere are taken with all the parameters same then hollow will cool down at fast rate.
- Rate of temperature fall $R_F \propto \frac{1}{s} \propto \frac{dT}{dt}$,
  so $dt \propto s$. If condition of specific heat is $s_1 > s_2 > s_3$
  and if all cooled same temperature i.e. temperature fall is also identical for all then required time
  $\boxed{t \propto s}$
  $\therefore$ $t_1 > t_2 > t_3$

Illustration 51:

A body of $e = \frac{1}{3}$ at 127°C is kept in a room at 27°C. Calculate initial rate of heat loss if surface area of body is 10 cm² (Take $\sigma = 6 \times 10^{-8}$)

Solution:

$$\frac{dQ}{dt} = \frac{1}{3} \times 10 \times 10^{-4} \times 6 \times 10^{-8}\left[(400)^4 - (300)^4\right] = \frac{1}{3} \times 6 \times 10^{-12} \times 10\left[10^8(256-81)\right] = 0.35 \text{ Watt}$$

Illustration 52:

The operating temperature of a tungsten filament in an incandescent lamp is 2000 K and its emissivity is 0.3. Find the surface area of the filament of a 25-watt lamp. Stefan's constant $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4}

Solution:

$\because$ Rate of emission = wattage of the lamp

$\because$ $W = Ae\sigma T^4 \Rightarrow A = \frac{W}{e\sigma T^4} = \frac{25}{0.3\times5.67\times10^{-8}\times(2000)^4} = 0.918 \times 10^{-4}\ m^2$

Illustration 53:

Calculate the temperature at which a perfect black body radiates at the rate of 5.67 W cm^{-2}. Stefan's constant is 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}.

Solution:

Given $E = 5.67$ W $cm^{-2} = 5.67 \times 10^{+4}$ W m^{-2}, $\sigma = 5.67 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}

Using, $E = \sigma T^4$

$T^4 = \frac{E}{\sigma}$ or $T = \left[\frac{E}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{5.67 \times 10^{+4}}{5.67 \times 10^{-8}}\right]^{\frac{1}{4}} = (10^{12})^{1/4} = 1000$ K

Illustration 54:

A body of relative emissivity ($e_r = 0.80$), surface area of 12 cm^2 kept in thermal equilibrium in a room temperature 20°C. Calculate the amount of heat emitted by the body in 5 seconds is nearly. $\sigma = 6.0 \times 10^{-8}$ $W/m^2\text{-}K^4$.

Solution:

Given, Area $A = 12 \times 10^{-4}\ m^2$; Temperature, $T = 273 + 20 = 293$ K; Emissivity, $e = 0.80$

$Q = e\sigma AT^4 \times t \Rightarrow Q = 0.80 \times 6 \times 10^{-8} \times 12 \times 10^{-4} \times (293)^4 \times 5 = 2.12$ J

Illustration 55:

The rate of emission of radiation of a black body at 273°C is E, then the rate of emission of radiation of this body at 0°C will be: -

Solution:

$E = \sigma A(546)^4$ ...(i)

$E_{0°C} = \sigma A(273)^4$ ...(ii)

$\frac{E}{E_{0°C}} = 2^4 \Rightarrow E_{0°C} = \frac{E}{16}$

Illustration 56:

If temperature of ideal black body is decreased from T to $\frac{T}{2}$ than find out percentage loss in emissive rate.

Solution:

$E \propto T^4$, $E' \propto \left[\frac{T}{2}\right]^4 \propto \frac{T^4}{16}$

$\left[\frac{E-E'}{E}\right]\times100\% = \left[1-\frac{1}{16}\right]\times100\% = \frac{15}{16}\times100\% \approx 94\%$ Remaining is 6% (Approx)

Illustration 57:

For a black body at temperature 727°C, its radiating power is 60 watt and temperature of surrounding is 227°C. If temperature of black body is changed to 1227°C then its radiating power will be :-

Solution:

$$P = \frac{Q}{t} = \sigma A[T^4 - T_0^4]$$

$$P \propto (T^4 - T_0^4)$$

$$\frac{P_2}{P_1} = \frac{(1500)^4 - (500)^4}{(1000)^4 - (500)^4} = \frac{500^4(3^4 - 1)}{500^4(2^4 - 1)}$$

$$\frac{P_2}{60} = \frac{80}{15} \Rightarrow P_2 = 320 \text{ W}$$

Illustration 58:

Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The ratio of the energy radiated per second by the first sphere to that by the second is: -

Solution:

The energy radiated per sec is given by, $E = e\sigma AT^4$

for same material e is same, $\frac{E_1}{E_2} = \frac{T_1^4 A_1}{T_2^4 A_2} = \frac{T_1^4\left(4\pi r_1^2\right)}{T_2^4\left(4\pi r_2^2\right)} = \frac{(4000)^4 \times 1^2}{(2000)^4 \times 4^2} = \frac{1}{1}$

Illustration 59:

A sphere and cube made of same material and having same surface area are heated to the same temperature and kept in the same surrounding. The ratio of their initial rates of cooling will be: -

Solution:

$$R_f = \frac{e_r \sigma A\left[T^4 - T_0^{\,4}\right]}{ms} = \frac{e_r \sigma A\left[T^4 - T_0^{\,4}\right]}{V\rho S}$$

$$R_f \propto \frac{1}{V} \Rightarrow \frac{R_{f\,\text{sphere}}}{R_{f\,\text{cube}}} = \frac{a^3}{\frac{4}{3}\pi r^3}$$

$$\because 4\pi r^2 = 6a^2$$

$$\left(\frac{a}{r}\right) = \left(\frac{4\pi}{6}\right)^{1/2}$$

$$\frac{R_{f\,\text{sphere}}}{R_{f\,\text{cube}}} = \frac{1}{\frac{4}{3}\pi}\left(\frac{4\pi}{6}\right)^{3/2} \Rightarrow \sqrt{\frac{\pi}{6}} : 1$$

Illustration 60:

Two bodies A or B have same rate of cooling $\frac{m_A}{m_B} = \frac{3}{1}, \frac{s_A}{s_B} = \frac{1}{2}$. Find ratio of heat loss of A & B?

Solution:

Same rate of cooling

$$\left(\frac{eA\sigma\left(T_b^4 - T_s^4\right)}{m_A s_A}\right)_A = \left(\frac{eA\sigma\left(T_b^4 - T_s^4\right)}{m_B s_B}\right)_B \qquad R_H = \frac{dQ}{dt} = e\sigma A(T^4 - T_0^4)$$

$$\left(\frac{\text{Heat loss}}{3\times 1}\right)_A = \left(\frac{\text{Heat loss}}{1\times 2}\right)_B$$

$$2(\text{Heat loss})_A = 3(\text{Heat loss})_B \Rightarrow \frac{(\text{Heat loss})_A}{(\text{Heat loss})_B} = \frac{3}{2}$$

Illustration 61:

Two spheres A and B of different metals but radius are R and 2R kept at same temperature. The ratios of their masses and specific heats are 2 : 3 and 3 : 6. Which sphere will show the least rate of cooling (initially)?

Solution:

$$R_F = \frac{dT}{dt} = \frac{e\sigma A}{ms}(T^4 - T_0^4)$$

$$R_F \propto \frac{A}{ms} \propto \frac{r^2}{ms}$$

$$R_{F_A} : R_{F_B} = \left[\frac{1}{2\times 3}\right] : \left[\frac{2}{3\times 6}\right]$$

Newton's law of cooling

Rate of cooling $\left(\frac{dT}{dt}\right)$ is directly proportional to excess of temperature of the body over that of surrounding.

[(when $(T - T_0) \not> 35°C$]

$$-\frac{dT}{dt} \propto (T - T_0)$$

T = temperature of body [all temperatures in °C]

T_0 = temperature of surrounding,

$T - T_0$ = excess of temperature $(T > T_0)$

If the temperature of body is decreased by dT in time dt then rate of fall of temperature

$$-\frac{dT}{dt} = K(T - T_0)$$

Where negative sign indicates that the rate of cooling is decreasing with time and K is cooling constant.

Graphs of Newton's Law of Cooling

$$-\frac{dT}{dt} = K(T - T_0)$$

$$\frac{dT}{T - T_0} = -Kdt$$

Integrating both side

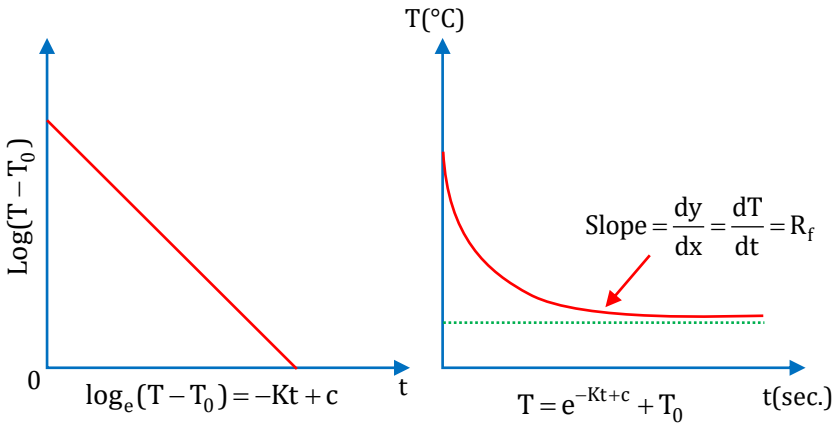
$$\int \frac{dT}{(T - T_0)} = -\int_0^t Kdt$$

$$\log_e (T - T_0) = -Kt + c$$

$$T - T_0 = e^{-Kt+c}$$

$$T = e^{-Kt+c} + T_0$$

$$y = e^{-x} + b$$



For Numerical Problems, Newton's Law of cooling

If the temperature of body decreases from T_1 to T_2 and temperature of surroundings is T_0 then average excess of temperature = $\left[\frac{T_1 + T_2}{2} - T_0\right]$

$$\left[\frac{T_2 - T_1}{t}\right] = -K\left[\frac{T_1 + T_2}{2} - T_0\right] \quad \text{or} \quad \left[\frac{T_1 - T_2}{t}\right] = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

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Limitations of Newton's Law of Cooling

- Temperature difference should not exceed 35° C, $(T - T_0) \not\leq 35°\ C$
- Newton's law of cooling is used to calculate the specific Heat of liquid.
- Loss of heat should only be by radiation.
- This law is an extended form of Stefan–Boltzmann's law.

Spectral Emissive Power and Wien's Displacement Law

Spectral Emissive Power (e_λ)

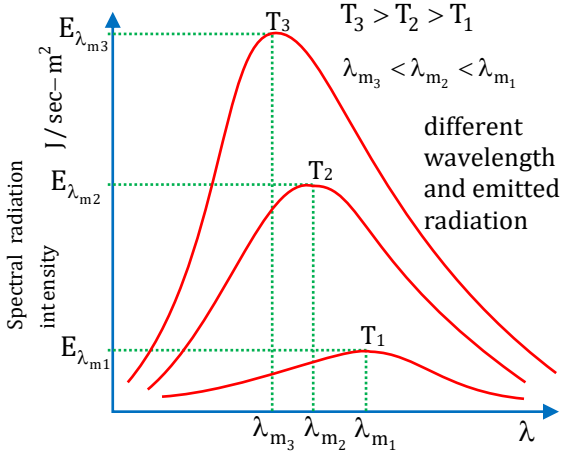
The amount of heat radiation emitted by unit area of the body in one second in unit spectral region at a given wavelength.

For GB, $e_\lambda = \frac{(Q_{\lambda\text{Å}})_{GB}}{At}$ For IBB, $E_\lambda = \frac{(Q_{\lambda\text{Å}})_{IBB}}{At}$

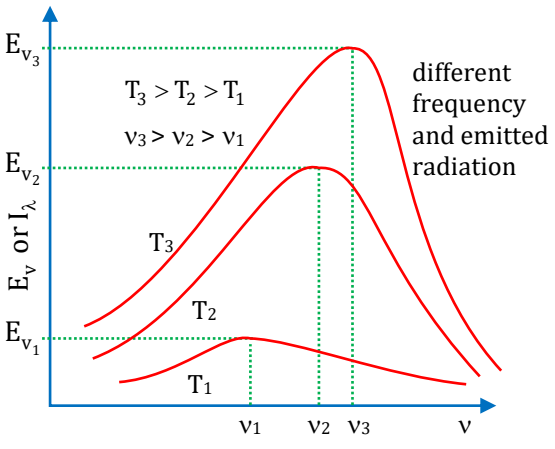
UNIT : SI $\rightarrow$ $W/m^2 - Å$

Spectral Energy distribution curve of Black Body radiations

Practically given by : Lumers and Pingshan



Mathematically given by : Plank



- At any temperature the spectral energy distribution curve for surface of an ideal black body is always continuous.
- $\text{Area} = \int_0^\infty E_\lambda d\lambda = E = \sigma T^4$

  $E \propto T^4 \therefore \text{ Area} \propto T^4$

  Hence, $\frac{A_1}{A_2} = \left[\frac{T_1}{T_2}\right]^4$
- If the temperature of a black body is increased, the wavelength corresponding to maximum emission of radiation is shifted from longer wavelength to shorter wavelength, due to this colour of the body changes.

  $T_3 > T_2 > T_1$; $\lambda_{m_3} < \lambda_{m_2} < \lambda_{m_1}$

Wein's Displacement Law

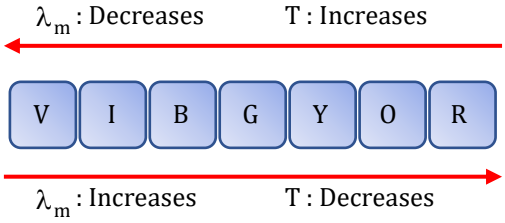
The wavelength (λ_m) corresponding to maximum emission of radiation decrease with increasing temperature $\left[\lambda_m \propto \frac{1}{T}\right]$ This is known as Wein's displacement law.

$\lambda_m \propto \frac{1}{T} \Rightarrow \lambda_m = \frac{b}{T}$

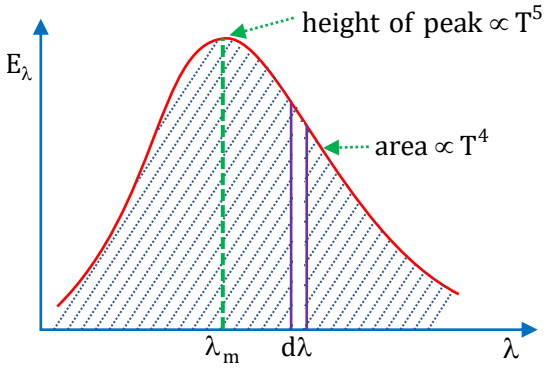
- For numerical $\Rightarrow \lambda_m T = b$ (where b is Wein's constant = 2.89 × 10^{-3} mK)

  $\boxed{\lambda_{m_1} T_1 = \lambda_{m_2} T_2}$

- Dimensions of b : $[M^{\circ}L^{1}T^{\circ}\ \theta^{1}]$
- Relation between frequency and temperature $\nu_m = \frac{c}{b}T \qquad [c = \lambda \times \nu]$
- $\left[\lambda_m \propto \frac{1}{T}\right]$



The area enclosed between spectral energy curve and wavelength axis is represented total Emissive power (E)



spectral energy distribution curve $(E_\lambda - \lambda)$

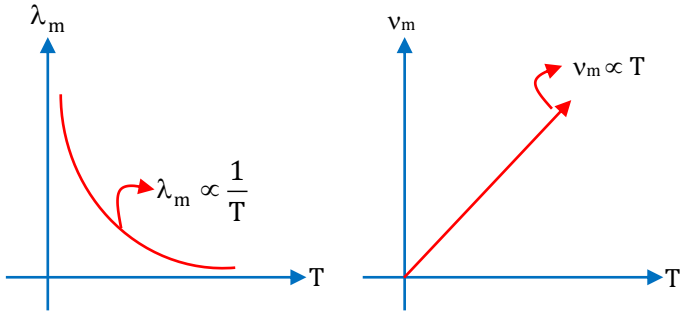
Results from these Graph

(i) Wein's 5th power law $\Rightarrow$ $E_{\lambda_m} \propto T^5$

(ii) i.e. Area $\int_0^\infty E_\lambda \, d\lambda = E = \sigma T^4$ $\Rightarrow$ Hence $\frac{A_1}{A_2} = \frac{E_1}{E_2} = \left[\frac{T_1}{T_2}\right]^4$

Area between curve and λ axis gives the emissive power of body $E = \frac{Q}{At} = \sigma T^4$

- Energy $= \frac{hc}{\lambda} = h\nu$



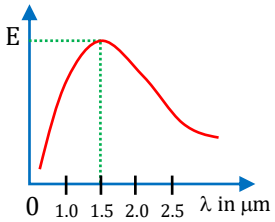
Key point: Spectral energy distribution curves are continuous. At any temperature in all possible wavelength radiation between (0 – ∝) are emitted but quantity of radiations is different for different wavelength.
As the wave length increases, the amount of radiation emitted first increase, becomes maximum and then decreases.

Illustration 62:
Temperatures of two stars are in ratio 3 : 2. If wavelength of maximum radiation from first body is 4000 Å, what is corresponding wavelength of second body ?

Solution:

$$\lambda_{max} \propto \frac{1}{T} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} \Rightarrow \frac{\lambda_2}{4000} = \frac{3}{2} \Rightarrow \lambda_2 = 6000\text{Å}$$

Illustration 63:
In the figure, the distribution of spectral energy of the radiation emitted by a black body at a given temperature is shown. The possible temperature of the black body is :- (b = 3×10^{-3} mk)



Solution:
P using Wiens law

$\lambda_m T = b$

$(1.5 \times 10^{-6})T = b$

$1.5 \times 10^{-6} T = 3 \times 10^{-3}$

$T = \frac{3\times10^{-3}}{1.5\times10^{-6}}$; $T = 2 \times 10^3$; $T = 2000$ K

Illustration 64:
Two bodies A and B have thermal emissivity's of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same, the two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy of B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation of A by 1.0 mm. If the temperature of A is 5802K, Calculate :- (a) The temperature of B (b) Wavelength λ_B

Solution:
(a) As both bodies A and B having same radiant power

$$\therefore P_A = P_B \Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4 \Rightarrow (0.01)\sigma A T_A^4 = (0.81)\sigma A T_B^4$$

$$T_B = \left(\frac{0.01}{0.81}\right)^{1/4} T_A = \frac{T_A}{3} = \frac{5802}{3} = \boxed{1934\ \text{K}}$$

(b) According to wein's displacement law

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A = 3\lambda_A$$

$$\text{As } \lambda_B - \lambda_A = 1\text{m} \Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1\text{m} \Rightarrow \frac{2\lambda_B}{3} = 1\text{m} \Rightarrow \lambda_B = 1.5\text{m}$$

Illustration 65:
Body A has $\lambda_{m_A} = 500\text{Å}$ and body B has $\lambda_{m_B} = 1000\text{Å}$.

Find : (a) $\frac{T_A}{T_B}$ (b) $\frac{\text{Emissive power of A}}{\text{Emissive power of B}}$

Solution:

(a) $\frac{T_A}{T_B} = \frac{\lambda_{m_a}}{\lambda_{m_b}} = \frac{1000\text{Å}}{500\text{Å}} = \frac{2}{1}$ (b) $\frac{\text{Emissive power of A}}{\text{Emissive power of B}} = \frac{\sigma T_A^4}{\sigma T_B^4} = \left(\frac{2}{1}\right)^4 = \frac{16}{1}$

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Solar Constant

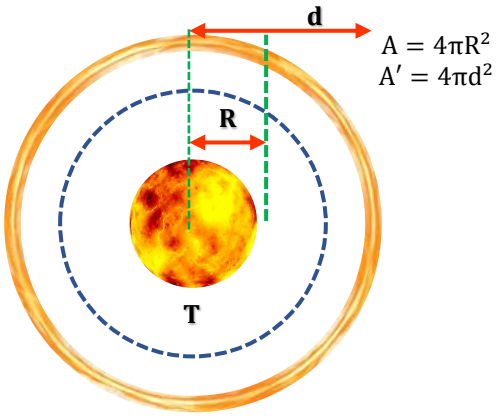
The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth.

The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.

The solar constant S is taken for earth to be 1340 watt/m^2

Temperature of the Sun

Let R be the radius of the Sun and 'd' be the radius of Earth's orbit around the Sun. Let E be the energy emitted by the Sun per second per unit area. The total energy emitted by the Sun in one second = E.A = $E \times 4\pi R^2$. (This energy is falling on a sphere of radius equal to the radius of the Earth's orbit around the Sun i.e., on a sphere of surface area $4\pi d^2$) So, the energy falling per unit time per unit area of Earth $= \frac{4\pi R^2 \times E}{4\pi d^2} = \frac{E R^2}{d^2}$



Let R be the radius of the Sun and 'd' be the radius of Earth's orbit around the Sun.

$R = 7\times10^8$m, $d = 1.5\times10^{11}$ m, $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}

Solar constant $S = \frac{E R^2}{d^2}$

By Stefan's Law $E = \sigma T^4$

$$S = \frac{\sigma T^4 R^2}{d^2} \Rightarrow T = \left[\frac{S \times d^2}{\sigma \times R^2}\right]^{\frac{1}{4}} = \left[\frac{1340 \times (1.5\times10^{11})^2}{5.7\times10^{-8} \times (7\times10^8)^2}\right]^{\frac{1}{4}} = 5732K$$

Note: $S = \frac{\sigma T^2 R^2}{d^2} \Rightarrow S \propto \frac{1}{d^2} \Rightarrow \frac{S_{planet}}{S_{earth}} = \left(\frac{d_e}{d_p}\right)^2$

Illustration 66:

Estimate the temperature of the surface of sun when average radius of the sun and earth are 7×10^5 Km and 1.5×10^8 Km respectively. The solar radiant power on the earth at noon time is 1400 W/m^2 and Stefan's Boltzmann constant is 5.67×10^{-8} W/m^2–K^4.

Solution:

$$S = \frac{\sigma T^4 R^2}{d^2}$$

[R is the radius of the Sun and 'd' is the radius of Earth's]

$$1400 = \frac{5.67\times10^{-8} \times T^4 \times 4\pi\left(7\times10^8\right)^2}{4\pi\times\left(1.5\times10^{11}\right)^2}$$

$$T^4 = \frac{3.96\times10^{26}}{3.49\times10^{11}} \Rightarrow T = 5802.5K$$

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BEGINNER'S BOX-4

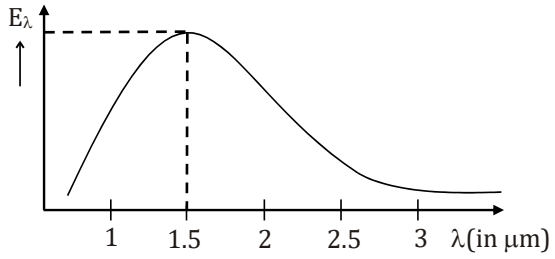
1. Explain why :
(a) a body with large reflectivity is a poor emitter.
(b) an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low value for the temperature of a red-hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace
(c) the earth without its atmosphere would be inhospitably cold
(d) Heat is generated continuously in an electric heater but its temperature remains constant after some time.
(e) The bottom of a cooking vessel is made black?
(f) On winter night you feel warmer when clouds cover the sky than when the sky is clear.
(g) A thermos or vacuum flask can keep hot things hot and cold things cold for a long time, how?

2. Two spherical ideal black bodies of radii r_1 and r_2 are having surface temperature T_1 and T_2 respectively, if both radiate the same power. Then calculate the ratio of $\frac{T_1}{T_2}$.

3. If a liquid takes 30 sec. in cooling from 80°C to 70°C and 70 sec in cooling from 60°C to 50°C, then find the room temperature.

4. A body cools in 10 minutes from 60°C to 40°C. What will be its temperature after next 10 minutes? The temperature of the surrounding is 10°C.

5. Calculate the temperature of the black body from given graph.



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Kinetic Theory of Gases

Introduction, Notations and Ideal Gas Concept

Kinetic Energy of Gas

All molecules of gas are continuously performing motion in random direction or say zig-zag direction. Apart from translational motion these molecules also perform rotational as well as vibrational motion. Thus, these molcules have kinetic energy associated with these motions.

Kinetic energy of a gas is defined as some of kinetic energies of all molecules.

It is dependent on absolute temperature. Increasing temperature increases kinetic energy of gas.

Potential Energy

Intermolecular attraction force exists between molecules and thus every molecules have some potential energy.

Greater Intermolecular force (IMF), lesser will be potential energy and greater will be magnitude of potential energy.

Orders

(i) Intermolecular force

Solid > Liquid > Gas (Real) > Ideal Gas (IMF = 0)

(ii) Potential Energy

Solid < Liquid < Gas (Real) < Ideal Gas (IMF = 0)

Internal Energy of Gas

Internal energy of gas is defined as some of potential energy and kinetic energy at a temperature.

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

Notation

Moles of gas $\mu = \frac{M}{M_W} = \frac{N}{N_a}$

Avogadro constant $N_a = 6.023 \times 10^{23} \ \frac{\text{molecules}}{\text{mole}}$

Universal gas constant $R = 8.31 \ \frac{J}{\text{mole-k}} \approx 2 \frac{\text{cal}}{\text{mole-k}}$

Specific gas constant $r = \frac{R}{M_W}$

Boltzmann constant $k = \frac{R}{N_a} = 1.38 \times 10^{-23} \ \frac{J}{\text{kelvin}}$

Mass of a gas molecule = m

Mass of gas = M

Molecular weight = M_w

No. of gas molecules = N

Molecular density $n = \frac{N}{V}$

Gas density $\rho = \frac{M}{V} = mn$

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The Kinetic Theory of Gases

Rudolph Claussius (1822–88) and James Clark Maxwell (1831–75) developed the kinetic theory of gases in order to explain gas laws in terms of the motion of the gas molecules. The theory is based on following assumptions regarding the motion of molecules and the nature of the gases.

Basic Postulates of Kinetic Theory of Gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those of another gas.
- The size is negligible in comparision to inter molecular distance (10^{-9} m).
- The gas molecules keep colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (i.e. the total energy before collision = total energy after the collision)
- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.
- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Ideal Gas

A gas which follows all gas laws and gas equation at every possible temperature and pressure is known as ideal or perfect gas.

Equation of State for Ideal Gas

$PV = \mu RT$ (μ = number of moles of gas)

$\Rightarrow \quad PV = \frac{M}{M_w}RT \quad \Rightarrow \quad PV = \left[\frac{mN}{mN_a}\right]RT$

$\Rightarrow \quad PV = \left[\frac{R}{N_a}\right]NT \quad \Rightarrow \quad PV = NkT \qquad \left(\text{Boltzmann constant } k = \frac{R}{N_a}\right)$

$\Rightarrow \quad P = \frac{\rho RT}{M_w} \qquad \left(\rho = \frac{M}{V}\right)$

Properties of Ideal Gas

- Ideal gas molecules can do only translational motion, so their kinetic energy is only translational kinetic energy.
- Ideal gas can not be liquified because IMF is zero.
- Specific heat of ideal gas is constant quantity and it does not change with temperature.
- All real gases behave as an ideal gas at high temperature and low pressure and low density.
- Gas molecules have point mass and negligible volume and velocity is very high (10^7 cm/s). That's why there is no effect of gravity on them.

NTP & STP

Properties	N.T.P. (Normal Temperature and Pressure)	S.T.P. (Standard Temperature and Pressure)
Temperature	0° C = 273.15 K	0.01° C = 273.16K
Pressure	1 atm	1 atm
Volume (1 mole)	22.4 litres	22.4 litres

1 atm = 1.01325×10^5 pascal

1 litre = 10^{-3} m^3 = 10^3 cm^3 = 10^3cc = 10^3ml

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Illustration 67:
Calculate volume of 6 moles of CO_2 gas at NTP?
Solution:
Volume of 1 mole CO_2 gas at NTP = 22.4 litres
Volume of 6 mole CO_2 gas at NTP = 6 × 22.4 litres = 134.4 litres
Illustration 68:
Calculate pressure exerted by 3 moles of an ideal gas kept in a vessel of volume 6 litres at 27°C?
Solution:
Given : V = 6 litres ; T = 300 K ; μ = 3 moles

$PV = \mu RT$

$$P = \frac{\mu RT}{V} \quad \Rightarrow \quad P = \frac{3 \times 0.0821 \times 300}{6} \quad \Rightarrow \quad P = 12.315 \text{ atm}$$

Gas Laws

An ideal gas is described by following terms or say following properties Temperature (T), Pressure(P), Volume of the gas(V) and Number of moles(μ). Relations between these properties for an ideal gas have been established and are know as gas laws.

(1) Charle's Law (2) Boyle's Law
(3) Gay-Lussac's Law (4) Avogadro's Law
(5) Dalton's Partial Pressure Law

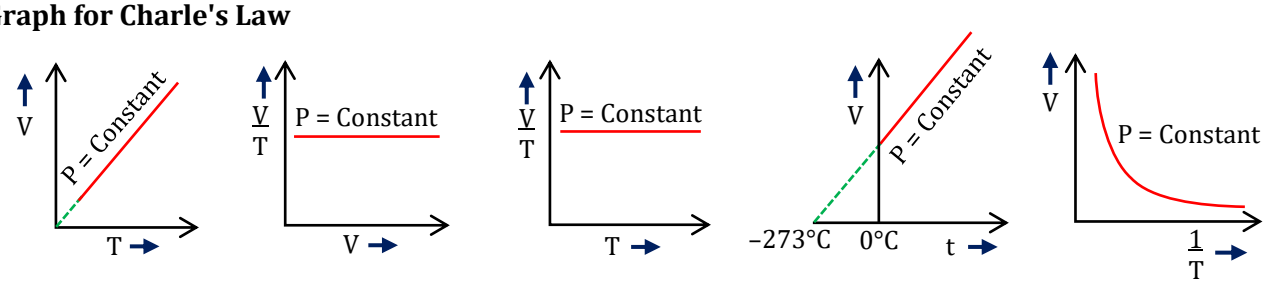
Charle's Law

According to this law, for a given mass of an ideal gas at constant pressure; volume of a gas is directly proportional to its absolute temperature,
Mathematically

$V \propto T$ (m and P are Constant)

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Graph for Charle's Law



Here t represents temperature in °C and T represents in K i.e. absolute temperature.

Boyle's Law

According to this law, for a given mass of an ideal gas at constant temperature; the volume of a gas is inversely proportional to its pressure,
Mathematically

$$V \propto \frac{1}{P}$$ (m and T are Constant)

$P_1V_1 = P_2V_2$

Graphs for Boyle's Law

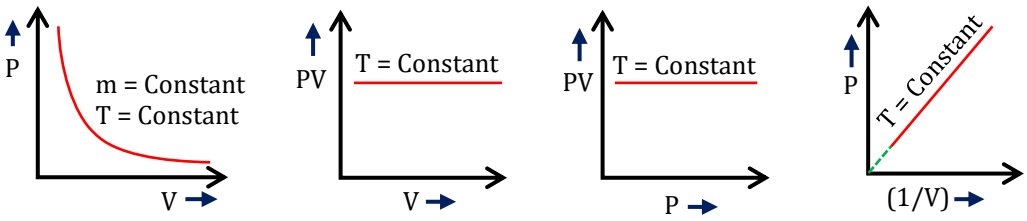


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Illustration 69:

At a constant pressure, N_2 occupies volume of 30 litres at 300 K. Find volume occupied by N_2 at 427°C.

Solution:

From Charle's Law, $V \propto T$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{30}{300} = \frac{V_2}{273+427} \quad \Rightarrow \quad V_2 = \frac{1}{10} \times 700 = 70 \text{ litres}$$

Illustration 70:

20 litres CH_4 exerts a pressure of 80 bar at a constant temperature. Find pressure exerted by the gas of volume 40 litres.

Solution:

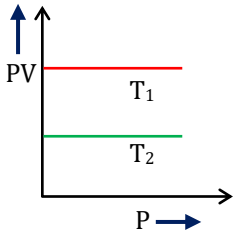
From Boyle's law,

$P_1V_1 = P_2V_2$

$20 \times 80 = P_2 \times 40 \quad \Rightarrow \quad P_2 = 40$ bar

Illustration 71:

Find relation between T_1 and T_2 for the given curve.



Solution:

From Boyle's Law $T_1 > T_2$

Gay–Lussac's Law

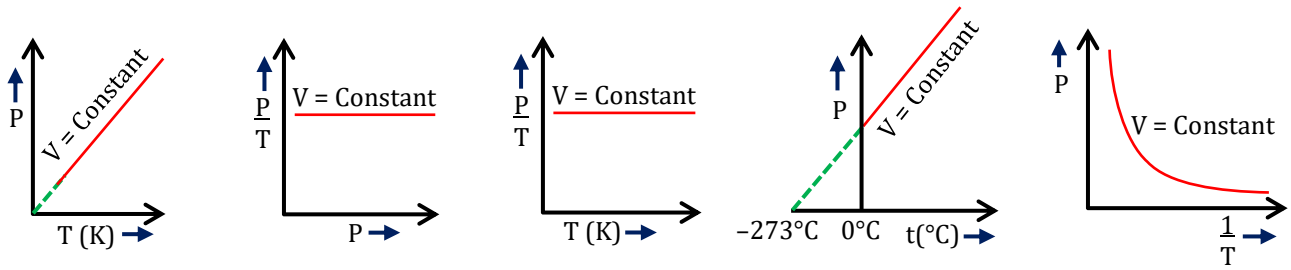
According to this law, for a given mass of an ideal gas; at constant volume, pressure of a gas is directly proportional to its absolute temperature,

Mathematically

$P \propto T$ (m and V are Constant)

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Graph for Gay–Lussac's Law



Here t represents temperature in °C and T represents in K i.e. absolute temperature.

Avogadro's Law

According to this law, at same temperature and pressure, equal volume of all gases contain equal number of molecules, i.e., $N_1 = N_2$ if P, V and T are same.

Dalton's Partial Pressure Mixture Law

According to this law, the pressure exerted by mixture of non-reactive gases is equal to the sum of partial pressure of each component gases present in the mixture.

$P = P_1 + P_2 + P_3 \ldots$

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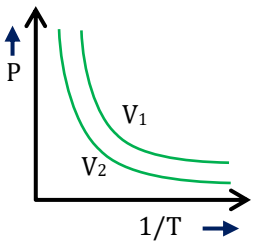
Illustration 72:

Find relation between V_1 and V_2 for given curve.



Solution:

From Gay–Lussac's Law, $V_1 < V_2$

Illustration 73:

At a particular temperature, N_2 exerts a pressure of 60 bar and O_2 exerts a pressure of 25 bar on the wall of closed vessel. Find pressure exerted by them on the wall if mixed together.

Solution:

From Dalton's Partial Pressure Mixture Law, $P_{net} = P_1 + P_2$

where $P_1 \rightarrow$ pressure exerted by N_2 alone = 60 bar

$P_2 \rightarrow$ pressure exerted by O_2 alone = 25 bar

$\therefore P_{net} = 60 + 25 = 85$ bar

Illustration 74:

At a particular temperature and pressure, 4 moles of N_2 occupy a volume of 30 litres. Then find volume occupied by 16 moles of O_2 at same temperature and pressure.

Solution:

From Avogadro's Law

$V \propto N$

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \Rightarrow \frac{30}{4} = \frac{V_2}{16} \Rightarrow V_2 = 120 \text{ L}$$

BEGINNER'S BOX-5

1. A vessel of volume 8.3×10^{-3} m^3 contains an ideal gas at temperature 27°C and pressure 200 kPa. The gas is allowed to leak till the pressure falls to 100 kPa and temperature increases to 327°C. What is the amount of gas in moles will be leaked out?

2. Two closed vessels of equal volume contain air at 105 kPa, 300 K and are connected through a narrow tube. If one of the vessels is now maintained at 300 K and other at 400 K, what will be the pressure in the vessels?

3. The volume of a gas is 1 litre at the pressure 1.2×10^7 Nm^{-2} and temperature 400 K. Calculate the number of molecules in the gas.

4. A vessel contains two non- reactive gases: neon (monatomic) and oxygen (diatomic) The ratio of their partial pressures is 3:2. Estimate the ratio of

(a) number of molecules and

(b) mass density of neon and oxygen in the vessel.

Atomic mass of Ne = 20.2 u. molecular mass of O_2 = 32.0 u.

Different Types of Speeds of Gas Molecules

Average velocity

Because molecules are in random motion in all possible direction with all possible velocity. Therefore, the average velocity of the gas molecules in container is zero.

$$<\vec{v}> = \frac{\vec{v}_1 + \vec{v}_2 + ...\vec{v}_N}{N} = 0$$

Average speed or Mean speed (v_{avg})

By Maxwell's velocity distribution law v_M or $<|\vec{v}|> \equiv v_{mean} = v_{avg}$

$$<|\vec{v}|> \equiv v_{mean} \equiv v_{avg} = \frac{|\vec{v}_1| + |\vec{v}_2| + ...|\vec{v}_n|}{N}$$

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Most probable speed (v_{mp})

At a given temperature, the speed to which maximum number of molecules belongs is called as most probable speed (v_{mp})

RMS Speed (v_{rms})

RMS Speed (root mean squared speed) is equal to square root of the sum of square of speed of each molecule of a gas.

$$\vec{v}_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 .....}{N}}$$

Illustration 75:

4 gas molecules have speeds 3u, 5u, 2u and 6u. Find their rms speed.

Solution:

$$v_{rms} = \sqrt{\frac{(3u)^2 + (5u)^2 + (2u)^2 + (6u)^2}{4}} = \sqrt{\frac{(9+25+4+36)u^2}{4}} = \sqrt{\frac{37}{2}}u$$

Illustration 76:

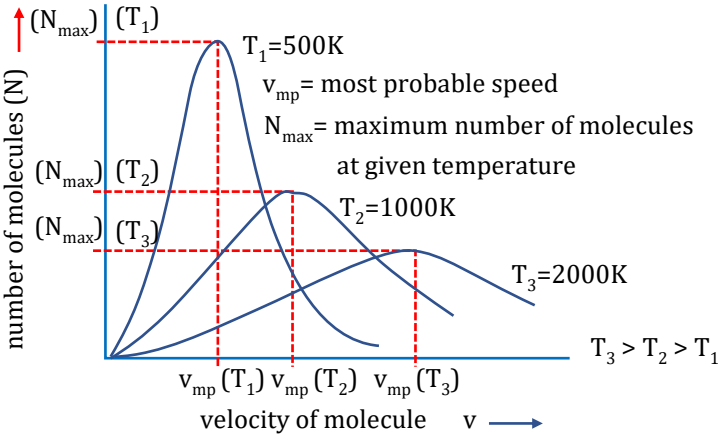
For above question find average speed.

Solution:

$$v_{avg} = \frac{3u+5u+2u+6u}{4} = 4u$$

Maxwell's law of distribution of velocities

Maxwell had drawn a curve between number of molecules, moving with a particular velocity, and velocity of molecules known as velocity distribution curve shown here.



[According to this law]

- The velocities of molecules of a gas are in between zero and infinity (0 – ∞).
- The area of the graph with velocity axis gives as total number of molecules available in the container which remains constant always.
- With the increase in the temperature, the most probable velocity increases but corresponding number of molecules decreases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- According to Maxwell

$$v_{rms} > v_{mean} > v_{mp}$$

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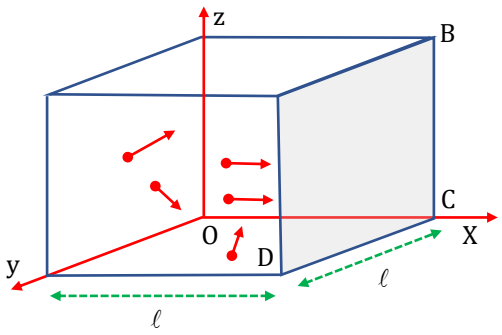
Illustration 77:

If for a gas, their most probable speed at temperature T_1 is v, at temperature T_2 is $\frac{3v}{2}$ and at temperature T_3 is $\frac{2v}{3}$. Find order of temperature.

Solution:

As we know most probable speed increases with increase in temperature. Hence $T_3 < T_1 < T_2$.

Expression for Pressure of An Ideal gas



Consider an ideal gas enclosed in a cubical vessel of length ℓ.

Suppose there are 'N' molecules in a gas which are moving with velocities $\vec{v}_1, \vec{v}_2 ..... \vec{v}_N$.

If we consider any single molecule then its instantaneous velocity $\vec{v}$ can be expressed as $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Due to random motion of the molecule

$v_x = v_y = v_z$; $|\vec{v}| = v_x\sqrt{3} = v_y\sqrt{3} = v_z\sqrt{3} = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Suppose a molecule of mass m is moving with a velocity v_x towards the face ABCD. It strikes the face of the cubical vessel and returns back to strike the opposite face.

Change in momentum of the molecule per collision $\Delta p = -mv_x - mv_x = -2mv_x$

Momentum transferred to the wall of the vessel per molecule per collision $\Delta p = 2mv_x$

The distance travelled by the molecule in going to face ABCD and coming back is 2ℓ.

So, the time between two successive collision is $\Delta t = \frac{2\ell}{v_x}$

Number of collision per sec per molecule is $f_c = \frac{v_x}{2\ell} = \frac{\text{molecule velocity}}{\text{mean free path}}$; $f_c = \frac{v_{rms}}{\lambda_m}$ or $f_c = \frac{v_m}{\lambda_m}$

Hence momentum transferred to the wall per second by the molecule is equal to force applied on the wall therefore, force

$$F = (2mV_x)\frac{v_x}{2\ell} = \frac{mv_x^2}{\ell} = \frac{mv^2}{3\ell} \quad \left[\text{As } V = \sqrt{3}V_x\right]$$

Pressure exerted by gas molecule $P = \frac{F}{A} = \frac{1}{3}\frac{mv^2}{\ell \times A} \Rightarrow P = \frac{1}{3}\frac{mv^2}{V} \quad [\because A \times \ell = V]$

Pressure exerted by gas $P = \sum\frac{1}{3}\frac{mv^2}{V} = \sum\frac{1}{3}\frac{mv^2}{V} \times \frac{N}{N} = \frac{1}{3}\frac{mN}{V}\frac{\sum v^2}{N} = \frac{1}{3}\frac{mN}{V}v_{rms}^2$

$$\Rightarrow v_{rms}^2 = \frac{3PV}{M} = \frac{3\mu RT}{\mu M_w} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M_w}}, P = \frac{1}{3}\frac{M}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$$

- Average number of molecules for each wall $= \frac{N}{6}$.

  No. of molecules along each axis $= \frac{N}{3}$ $(N_x = N_y = N_z)$

- $\bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{v_{rms}^2}{3}$ Root mean square velocity along any axis for gas molecule is

  $(v_{rms})_X = (v_{rms})_y = (v_{rms})_z = \frac{v_{rms}}{\sqrt{3}}$

All gas laws and gas equation can be obtained by expression of pressure of gas (except Joule's law)

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Translational Kinetic Energy

$$E_T = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + ...$$

$$E_T = \frac{1}{2}m[v_1^2 + v_2^2 + v_3^2 + ...] \times \frac{N}{N}$$

$$E_T = \frac{1}{2}MV_{rms}^2 \qquad \left\{P = \frac{1}{3}\rho(V_{rms})^2 \Rightarrow (V_{rms})^2 = \frac{3P}{\rho}\right\}$$

$$E_T = \frac{1}{2}M\left(\frac{3P}{\rho}\right) \qquad \left\{\rho = \frac{M}{V} \Rightarrow V = \frac{M}{\rho}\right\}$$

$$E_T = \frac{3}{2}PV$$

$\frac{E_T}{V} = E_v$ = Energy density

$$E_V = \frac{3}{2}P$$

Expression of RMS Speed

$$\Rightarrow \quad P = \frac{1}{3}\rho(V_{rms})^2 \quad \Rightarrow \quad V_{rms} = \sqrt{\frac{3P}{\rho}} \quad \Rightarrow \quad \frac{P}{\rho} = \frac{RT}{M_W} = \frac{K_bT}{m}$$

$$\Rightarrow \quad P = \frac{1}{3}\rho(V_{rms})^2 \quad \Rightarrow \quad V_{rms} = \sqrt{\frac{3P}{\rho}} \quad \Rightarrow \quad \frac{P}{\rho} = \frac{RT}{M_W} = \frac{k_bT}{m}$$

$$\Rightarrow \quad V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3k_bT}{m}}$$

If gas is same (M_w= const.) and temp. is changed $\Rightarrow$ $V_{rms} \propto \sqrt{T}$

If temperature is constant and gas is changed $\Rightarrow$ $V_{rms} \propto \frac{1}{\sqrt{M_w}}$

Effect of pressure at constant temp.:

$$V_{rms} \propto \frac{1}{\sqrt{M_w}} \quad \Rightarrow \quad P = \frac{\rho RT}{M_w}$$

$\because$ T & M_w = constant $\quad \therefore \frac{P}{\rho} = \text{const.}$

So, no effect on V_{rms}.

Note: If only temp. is changed $V_{rms} = \sqrt{\frac{3RT}{M_W}}$

If only pressure is changed $V_{rms} = \sqrt{\frac{3P}{\rho}}$

If both pressure & temp. is changed $V_{rms} = \sqrt{\frac{3RT}{M_W}}$

Most probable speed

$$v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2k_bT}{m}}$$

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Average speed

$$v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8k_bT}{\pi m}}$$

Comparision

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3k_bT}{m}} \qquad \& \qquad v_{av} = \sqrt{\frac{8P}{\pi\rho}} = \sqrt{\frac{8RT}{\pi M_w}} = \sqrt{\frac{8k_bT}{\pi m}}$$

$$v_{mp} = \sqrt{\frac{2P}{\rho}} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2k_bT}{m}}$$

$$\boxed{V_{rms} > V_{Mean} > V_{MP}}$$

Escape Velocity

The minimum velocity required for which molecule or object to escape out from earth surface.

$$V_{es} = \sqrt{\frac{2GM}{R}} \qquad \Rightarrow \qquad (V_{es})_{earth} = 11.2 km/sec$$

Relation of escape Velocity with rms

Condition for atmosphere to be present at the surface of any planet:

$V_{rms} < V_{es} \rightarrow$ Atmosphere present

$V_{rms} \geq V_{es} \rightarrow$ No atmosphere

$$\sqrt{\frac{3RT}{M_W}} \geq V_{es} \qquad \Rightarrow \qquad T \geq \frac{(V_{es})^2 M_w}{3R} \qquad \Rightarrow \qquad T_{min} = \frac{(V_{es})^2 M_w}{3R}$$

$$T_{min} = \frac{M_w}{2} \times 10^7 K \qquad \rightarrow \qquad \text{here } M_w \text{ is in kg}$$

$$T_{min} = \frac{M_w}{2} \times 10^4 K \qquad \rightarrow \qquad \text{here } M_w \text{ is in gram}$$

Note: If we continuously increase the temperature at 10^4 K hydrogen gas molecules will leave the earth surface first.

Moon has no atmosphere because there rms speed of gas molecules is greater then the escape velocity.

Illustration 78:

The rms velocity of gas molecules of a given amount of a gas at 27°C and 1.0×10^5 N m^{-2} pressure is 100 m sec^{-1}. If temperature and pressure are respectively 527°C and 2.5×10^5 N m^{-2}, the rms velocity will be: -

Solution:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

For a given gas, $v_{rms} \propto \sqrt{T}$

$$\frac{v_{rms_1}}{\sqrt{T_1}} = \frac{v_{rms_2}}{\sqrt{T_2}} \qquad \Rightarrow \qquad \frac{100}{\sqrt{300}} = \frac{v_{rms_2}}{\sqrt{800}} \qquad \Rightarrow \qquad v_{rms_2} = 163 m/s$$

Illustration 79:

The rms velocity of H_2 is 8×10^3 m /s. What will be the rms velocity of O_2 molecules at the same temperature :-

Solution:

$$v_{r.m.s.} \propto \sqrt{\frac{1}{M_w}} \qquad \Rightarrow \qquad \frac{(v_{r.m.s.})_{O_2}}{(v_{r.m.s.})_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4} \qquad \Rightarrow \qquad (v_{r.m.s.})_{O_2} = \frac{1}{4} \times 8 \times 10^3 = 2 \times 10^3 \text{ m/s}$$

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Illustration 80:
At a constant temperature if pressure of a gas is tripled. Find fractional change in most probable speed.

Solution:

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}$$

At constant temperature $\frac{P}{\rho}$ is constant hence v_{mp} does not change by change pressure.

Illustration 81:
Calculate the minimum temperature at which N_2 molecules escape from the earth.

Solution:

$$v_{rms} = \sqrt{\frac{3RT}{M_w}} \geq 11.2\times10^3 \text{ m/s} \Rightarrow \quad T \geq \left(11.2\times10^3\right)^2 \times \frac{M_w}{3R}$$

For minimum temperature $\quad T = \left(11.2\times10^3\right)^2 \times \frac{M_w}{3R} = 14 \times 10^7 \text{K}$

Degrees of Freedom (f)
The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called its degrees of freedom.

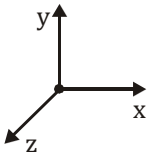
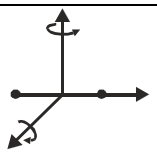
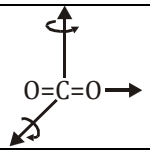
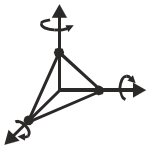
The number of independent coordinates required to specify the dynamic state of a system is called its degrees of freedom.

Types of Degrees of freedom
(a) Translational Degree of freedom : Number of independent ways in which a gas molecule may perform translatory motion. There are maximum three degree of freedom corresponding to translational motion.

(b) Rotational Degree of freedom : Number of independent ways in which a gas molecule may perform rotational motion. The number of degrees of freedom in this case depends on the structure of the molecule.

(c) Vibrational Degree of freedom : Number of independent ways in which a gas molecule may vibrate. It is exhibited at high temperature only.

Degrees of freedom for different gases according to atomicity of gas at low temperature

Atomicity of gas	Translational	Rotational	Total	
Monoatomic **Ex.** Ar, Ne, Ideal gas etc	3	0	3	
Diatomic **Ex.** O_2, Cl_2, N_2 etc.	3	2	5	
Triatomic (linear) **Ex.** CO_2, C_2H_2	3	2	5	
Triatomic (Non–linear) or Polyatomic **Ex.** H_2O, NH_3, CH_4	3	3	6	

At high temperature a diatomic molecule has 7 degrees of freedom. (3 translational, 2 rotational and 2 vibrational)

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Note: Degrees of freedom depends on atomicity, structure & temperature of gases.
Total number of degree of freedom = 3N
Where N = Atomicity of gas molecule
For a gas molecule
Vibrational DoF = Total DoF – (Translational + Rotational) DoF

Example: Diatomic gas molecule like O_2, Cl_2, N_2 (Atomicity N = 2)
Translational DoF = 3 ; Rotational DoF = 2 ; Vibrational DoF = 1
Total DoF = 3 + 2 + 1 = 6 = 3×2

Example: Triatomic linear gas molecule like CO_2 (Atomicity N = 3)
Translational DoF = 3 ; Rotational DoF = 3 ; Vibrational DoF = 3
Total DoF = 3 + 3 + 3 = 9 = 3×3

Maxwell's law of equipartition of energy

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

For one molecule of gas

Energy related with each degree of freedom = $\frac{1}{2}kT$

Energy related with all degree of freedom = $\frac{f}{2}kT$

$\because \quad \bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2 = \frac{v_{rms}^2}{3} \quad \Rightarrow \quad \frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

So, energy related with one degree of freedom = $\frac{1}{2}m\frac{v_{rms}^2}{3} = \frac{3}{2}\frac{kT}{3} = \frac{1}{2}kT$

Gas	f	Translational	Rotational	Total
Monoatomic	3	$\frac{3}{2}k_bT$	0	$\frac{3}{2}k_bT$
Diatomic	5 (3+2)	$\frac{3}{2}k_bT$	$\frac{2}{2}k_bT$	$\frac{5}{2}k_bT$
Triatomic Or Polyatomic (Linear)	5 (3+2)	$\frac{3}{2}k_bT$	$\frac{2}{2}k_bT$	$\frac{5}{2}k_bT$
Triatomic Or Polyatomic (Non-Linear)	6 (3+3)	$\frac{3}{2}k_bT$	$\frac{3}{2}k_bT$	$\frac{6}{2}k_bT$

Note: For ideal gas if atomicity is not known then take f = 3

Illustration 82:

Find rotational energy of 4 moles of N_2 at 127°C.

Solution:

T = 400 K & μ = 4

Rotational energy of N_2 (Diatomic gas) = $\frac{2}{2}\mu RT = 4 \times 8.314 \times 400 = 13302$ J

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Illustration 83:

At 127°C temperature, the total energy of an ideal gas is E_1. If the temperature is increased to 527°C, then total energy would be

Solution:

Total energy $E \propto T$

$$\frac{E_1}{T_1} = \frac{E_2}{T_2} \quad \Rightarrow \quad \frac{E_1}{400} = \frac{E_2}{800} \quad \Rightarrow \quad E_2 = 2E_1$$

Illustration 84:

Find ratio of total energy of 4 moles of monoatomic gas at 300 K and 6 moles of diatomic gas at 280 K.

Solution:

$$\text{Ratio} = \frac{4 \times \frac{3}{2} R \times 300}{6 \times \frac{5}{2} R \times 280} = \frac{3}{7}$$

Illustration 85:

7 moles of triatomic gas is at 400 K then what fraction of total energy is rotational energy?

Solution:

$$\text{Kinetic energy} = \frac{3}{2}\mu RT \quad ; \quad \text{Rotational energy} = \frac{2}{2}\mu RT \quad ; \quad \text{Total energy} = \frac{3}{2}\mu RT + \frac{2}{2}\mu RT = \frac{5}{2}\mu RT$$

$$\text{Fraction} = \frac{\frac{2}{2}\mu RT}{\frac{5}{2}\mu RT} = \frac{2}{5}$$

Molar Specific Heat of gas and Mayer's law

Specific Heat

The amount of energy required to raise the temperature of unit mass of a substance by 1°C (or 1K) is called its specific heat. It is represented by s or c.

If the temperature of a substance of mass m changes from T to T + dT when it exchanges an amount of heat dQ with its surroundings then its specific heat is $c = \frac{1}{m}\frac{dQ}{dT}$

Specific heats can be molar or gram.

(i) Gram specific heat

The amount of energy required to raise the temperature of 1 gram of a substance by 1°C (or 1K) is called its specific heat.

If Q amount of heat is required to raise the temperature of mass m (in gram) of a substance by temperature ΔT, then molar specific heat of the gas is given by,

$$c_{gram} = \frac{1}{m}\frac{Q}{\Delta T} \qquad \text{Unit : Joule/kg–°C, cal/g –°C}$$

(ii) Molar specific heat

The amount of energy required to raise the temperature of 1 mole of a substance by 1°C (or 1K) is called its specific heat.

If Q amount of heat is required to raise the temperature of μ moles of a substance by temperature ΔT, then molar specific heat of the gas is given by,

$$c_{molar} = \frac{1}{\mu}\frac{Q}{\Delta T} \qquad \text{Unit : Joule/mol–°C, cal/mol –°C}$$

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Note: Value of specific heats of gas can vary from zero (0) to infinity.

- Generally, two types of specific heats are defined for a gas –
  (a) Specific heat at constant volume (c_V)
  (b) Specific heat at constant pressure (c_P)
- There are many possible processes to give heat to a gas.
  A specific heat can be associated to each such process which depends on the nature of process.

Molar Specific of gas for different processes

(i) Isochoric Process V = Constant ; $c_V = \frac{Q}{\mu \Delta T}$

(ii) Isobaric Process P = Constant ; $c_P = \frac{Q}{\mu \Delta T}$

(iii) Isothermal Process T = Constant ; $c = \frac{Q}{\mu \Delta T}$

$\therefore \Delta T = 0 \Rightarrow c_{isothermal} = \infty$

(iv) Adiabatic Process Q = 0 ; $c = \frac{Q}{\mu \Delta T} \Rightarrow c_{adiabatic} = 0$

Mayer's Law

For an ideal gas $C_v = \frac{f}{2}R$; $C_v = \left(\frac{f}{2}+1\right)R$

$C_p - C_v = R$ (this is known as Mayer's law)

If gram specific heats are given :

$c_{molar} = M_w (c_{gram})$ where, M_w is molar mass of the gas

So, $(C_p)_{molar} = M_w(c_p)_{gram}$, $(C_v)_{molar} = M_w(C_v)_{gram}$

By Mayer's Law,

$C_p - C_v = R \Rightarrow M_w(c_p)_{gram} - M_w(c_V)_{gram} = R \Rightarrow (c_p)_{gram} - (c_V)_{gram} = \frac{R}{M_w}$

Gas	f	c_V	c_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} \approx 1.67$
Diatomic	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} \approx 1.4$
Triatomic Or Polyatomic (Linear)	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} \approx 1.4$
Triatomic Or Polyatomic (Non-Linear)	6	$\frac{6}{2}R$	$\frac{8}{2}R$	$\frac{4}{3} \approx 1.33$

Molar specific heat for mixture of gases

Suppose we have two ideal gases of moles μ_1 & μ_2 with specific heats c_1 & c_2 respectively. Now if they are mixed and then heated till their temperature changes by ΔT.

If $Q_1 \rightarrow$ heat required to change the temperature of ideal gas A by ΔT

$Q_2 \rightarrow$ heat required to change the temperature of ideal gas B by ΔT

$Q_{mix} \rightarrow$ total heat required to change the temperature of mixture by ΔT

then from energy conservation

$Q_{mix} = Q_1 + Q_2$

$\Rightarrow \mu_{mix} c_{mix} \Delta T = \mu_1 c_1 \Delta T + \mu_2 c_2 \Delta T$

$\Rightarrow c_{mix} = \frac{\mu_1 c_1 + \mu_2 c_2}{\mu_1 + \mu_2}$ ($\because \mu_{mix} = \mu_1 + \mu_2$)

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If c_v of individual gases are given then $(c_V)_{mix} = \frac{\mu_1 c_{V_1} + \mu_2 c_{V_2}}{\mu_1 + \mu_2}$

If c_P of individual gases are given then $(c_P)_{mix} = \frac{\mu_1 c_{P_1} + \mu_2 c_{P_2}}{\mu_1 + \mu_2}$

$$\gamma_{mix} = \frac{(c_P)_{mix}}{(c_V)_{mix}} = \frac{\mu_1 c_{P_1} + \mu_2 c_{P_2}}{\mu_1 c_{V_1} + \mu_2 c_{V_2}}$$

Illustration 86:
3 moles of monoatomic gas are heated at constant pressure from 27°C to 227°C. Find amount of heat required (in terms of R).

Solution:

At constant pressure molar specific of monoatomic gas $c = c_P = \frac{5}{2}R$

$\Delta T = 200$ K

Heat required $Q = \mu c_P \Delta T = 3 \times \frac{5}{2} R \times 200 = 1500R$

Illustration 87:
If 4 moles of diatomic gas are heated at constant volume by giving 300R heat. Then calculate change in temperature of the gas.

Solution:
At constant volume

$C_v = \frac{5}{2}R$ (for diatomic gas)

$\mu C_v \Delta T = Q \Rightarrow 4 \times \frac{5}{2} R \times \Delta T = 300R \Rightarrow \Delta T = 30$ K

Illustration 88:
If 4 moles of a monoatomic gas and 2 moles of a diatomic gas are mixed then find value of γ for the mixture.

Solution:

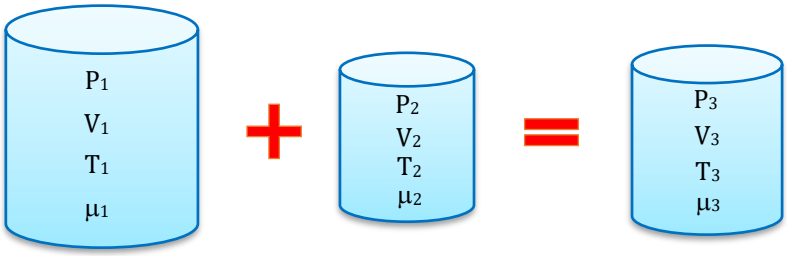
Monoatomic gas : $\mu_1 = 4$, $c_{V_1} = \frac{3}{2}R$, $c_{P_1} = \frac{5}{2}R$

Diatomic gas : $\mu_2 = 2$, $c_{V_2} = \frac{5}{2}R$, $c_{P_2} = \frac{7}{2}R$

$$\gamma_{mix} = \frac{\mu_1 c_{P_1} + \mu_2 c_{P_2}}{\mu_1 c_{V_1} + \mu_2 c_{V_2}} = \frac{4 \times \frac{5}{2}R + 2 \times \frac{7}{2}R}{4 \times \frac{3}{2}R + 2 \times \frac{5}{2}R} = 1.54$$

Mixture of Gases

Case – I
Here two gases kept in two different vessels are mixed in another vessel.



Here following result may or may not be possible.

$P_1 + P_2 \neq P_3$; $V_1 + V_2 \neq V_3$; $T_1 + T_2 \neq T_3$

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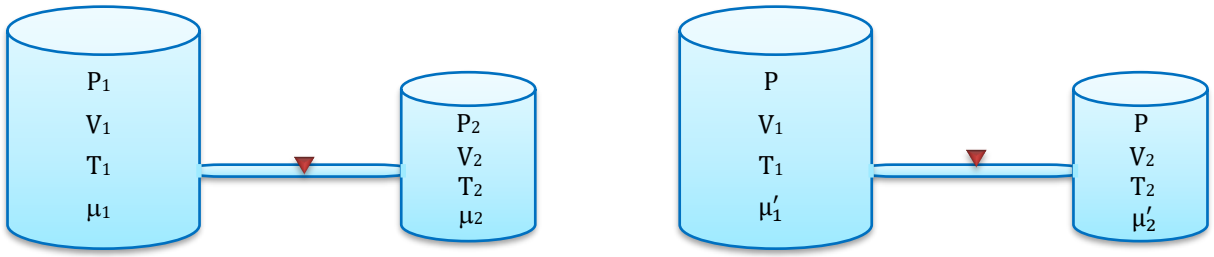
But we can surely say that

$\mu_1 + \mu_2 = \mu_3$ [From mass conservation]

$$\Rightarrow \quad \frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}=\frac{P_3V_3}{RT_3} \quad \Rightarrow \quad \frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}=\frac{P_3V_3}{T_3}$$

Case – II

Here two gases kept in two different vessels connected via a tube as shown. Cross-section area of this tube is very small so its volume can be neglected w.r.t. volume of vessel.



Initially the valve is closed which does not let mixing of gases but after opening of the valve gases will mix until pressure of both vessels become equal.

Also, we can say from mass conservation $\mu_1 + \mu_2 = \mu_1' + \mu_2'$

$$\Rightarrow \quad \frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}=\frac{PV_1}{RT_1'}+\frac{PV_2}{RT_2'} \quad \Rightarrow \quad \frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}=\frac{PV_1}{T_1'}+\frac{PV_2}{T_2'}$$

Illustration 89:

Two gases kept in two identical vessel of volume 30 litres at same temperature 300 K and same pressure 20 atm are mixed in a vessel having same volume V. The resulting pressure of the mixture will be :

Solution:

$$\frac{P_1V_1}{T_1}+\frac{P_2V_2}{T_2}=\frac{P_3V_3}{T_3} \quad \Rightarrow \quad \frac{P_1}{T_1}+\frac{P_2}{T_2}=\frac{P_3}{T_3}$$

$$\Rightarrow \quad \frac{20}{300}+\frac{20}{300}=\frac{P}{300} \quad \Rightarrow \quad P = 40 \text{ atm}$$

Illustration 90:

A vessel contains a mixture of Methane of mass 8g and Oxygen of mass 24g at temperature 300K and pressure 2atm. The density of the mixture is :-

Solution:

$$\mu_{mixture} = \frac{8}{16}+\frac{24}{16} = 2$$

$PV = \mu_{mixture} RT$

$\Rightarrow \quad 2V = 2 \times 0.0821 \times 300$

$\Rightarrow \quad V = 24.63$ litres

$$\rho = \frac{m}{V}$$

$$\rho = \frac{8+24}{24.63} = 1.3 \text{ g/litre}$$

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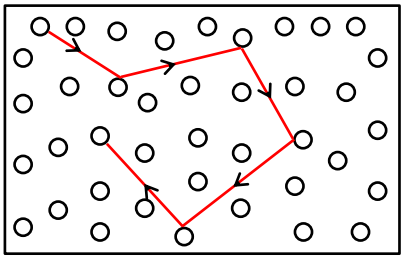
Mean Free Path and Real Gases

Mean Free Path

A molecule in its path undergoes a number of collisions so the path traversed by it is not a straight line but somewhat zigzag. Between two successive collisions a molecule travels in a straight line with uniform velocity. As motion is random thus the distance travelled by molecule between two successive collisions is not always equal.



The average distance travelled by a molecule between two successive collisions is called as mean free path (λ_m) of the molecule.

$$\lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}$$

Here, $n = \frac{N}{V}$ = No. of molecules per unit volume

d = diameter of a molecule

Real Gas

Practically, no gas is ideal at all. They show some deviation from ideal gas behaviour and are known as Real gas. Main cause of deviation is two postulates that have been assumed in Kinetic Theory of Gases :-

(i) Interaction force between gas molecules are considered negligible

(ii) Size of Gas molecules is considered negligible but practically they are not negligible.

Compressibility Factor

It is used to express the extent of deviation of a real gas from ideal gas behaviour. It is denoted by Z.

For a real gas, unlike ideal gas $PV_{real} \neq \mu RT$

Compressibility factor of a real gas is given by

$$Z = \frac{PV_{real}}{\mu RT}$$

Where, P $\rightarrow$ Pressure of real gas ; $\mu \rightarrow$ no. of moles of real gas

T $\rightarrow$ Temperature of real gas ; $V_{real} \rightarrow$ Volume of real gas

For an ideal gas of same mole μ at same temperature T and pressure P, we can write

$$PV_{ideal} = \mu RT \quad \text{then} \quad Z = \frac{PV_{real}}{PV_{ideal}} = \frac{V_{real}}{V_{ideal}}$$

Now, we will have three different cases of Z

Case-I : $Z = 1 \Rightarrow [V_{real} = V_{ideal}]$

Thus, real gas behaves as ideal gas.

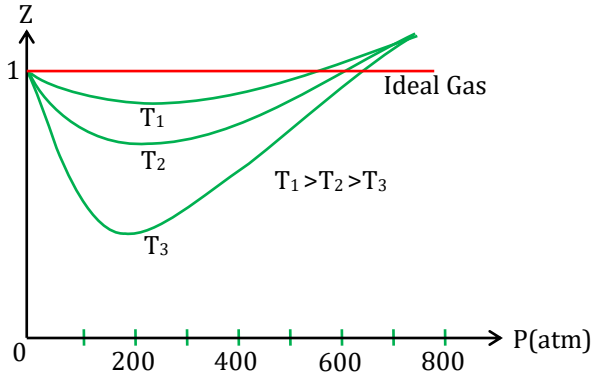
Case-II : $Z > 1 \Rightarrow [V_{real} > V_{ideal}]$

Thus, real gas expand with respect to ideal gas and hence we can say positive deviation.

Case-III : $Z < 1 \Rightarrow [V_{real} < V_{ideal}]$

Thus, real gas compress with respect to ideal gas and hence we can say negative deviation.

Compressibility Factor Curve



Thus, we can see from the curve that real gas approaches ideal gas behaviour at **Low Pressure** and **High Temperature**.

Vander Waal's Equation for Real Gas

Pressure Correction

Since, we have neglected intermolecular interaction, thus actual pressure of gas will be less than ideal pressure and it was found that

$$P_{ideal} = P_{real} + \frac{\mu^2 a}{V^2}$$

$a \rightarrow$ Constant and value depends on real gas

Volume Correction

Also, we have neglected size of molecules but actual volume occupied by real gas will be

$V_{ideal} = V_{real} - \mu b$

$b \rightarrow$ Constant and value depends on real gas

Now, applying ideal gas equations

$P_{ideal}V_{ideal} = \mu RT$

$$\left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$$

This equation is known as Van der Waal's Equation for Real Gas.

Illustration 91:

An oxygen molecule is travelling in air at 300 K and 1 atm, and the diameter of oxygen molecule is 1.2×10^{-10}m. Calculate the mean free path of oxygen molecule.

Solution:

$$n = \frac{N}{V} = \frac{P}{kT} = \frac{101.3\times10^3}{1.381\times10^{-23}\times300} = 2.449 \times 10^{25} \text{ molecules/m}^3$$

$d = 1.2 \times 10^{-10}$m

$$\lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}$$

$$\lambda = \frac{1}{\sqrt{2}\times\pi\times2.449\times10^{25}\times(1.2\times10^{-10})^2} \Rightarrow \lambda = \frac{1}{15.65\times10^5} \Rightarrow \lambda = 0.63 \times 10^{-6} \text{ m}$$

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Illustration 92:

A gas that is of 2 moles occupies a volume of about 500 ml at 300 Kelvin and 50 atmospheric pressure, calculate the compressibility factor of the gas.

Solution:

Compressibility factor $Z = \frac{PV_{real}}{\mu RT} = \frac{50 \times (500/1000)}{2 \times 0.082 \times 300} = 0.5081.$

Illustration 93:

Using van der Waals' equation, calculate the constant 'a' when two moles of a gas confined in a four-litre flask exerts a pressure of 11 atm at a temperature of 300 K. The value of 'b' is 0.05 litre mol^{-1}.

Solution:

Given v = 4L, P = 11atm, T = 300K, b = 0.05 litre/mol, μ = 2 moles

Vander Wall's equation for μ moles of gas is, $\left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$

Thus $\left(11 + a\frac{2^2}{4^2}\right)[4 - 2\,(0.05)] = 2 \times 0.082 \times 300 \Rightarrow a = 6.46$ atm litre^2 mol^{-2}

BEGINNER'S BOX-6

1. Given: Avogadro number N = 6.02×10^{23} molecules/mole and Boltzmann's constant k = 1.38×10^{-23} J/(molecule-K).

Calculate: -

(a) The average kinetic energy of translation of an oxygen molecule at 27°C

(b) The total kinetic energy of an oxygen molecule at 27°C

(c) The total kinetic energy of 1 mole of oxygen gas at 27°C.

2. The temperature of gas is –73°C. To what temperature should it be heated so that

(a) average kinetic energy of the molecule is doubled?

(b) the root mean square velocity of the molecules is doubled?

3. For a gas $\frac{R}{C_P} = 0.4$. For this gas calculate the following –

(a) atomicity and degree of freedom

(b) value of C_v and γ

(c) mean gram - molecular kinetic energy at 300 K temperature

4. If three molecules are having speeds v_1, v_2 and v_3 respectively, then what will be their average speed and root mean square speed?

5. Four molecules of a gas are having speeds of 1, 4, 8 and 16 ms^{-1}. Find the root mean square velocity of the gas molecules?

6. A flask contains argon and chlorine in the ratio of 2 : 1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of

(a) Average translational kinetic energy per molecule, and

(b) root mean square speed (v_{rms}) of the molecules of the two gases.

Atomic mass of argon = 39.9 u : Molecular mass of chlorine = 70.9 u.

Thermodynamics

Introduction

Branch of physics which deals with the inter–conversion between heat energy and any other form of energy is known as thermodynamics. In this branch of physics, we deal with the processes involving heat, work and internal energy. In this branch of science, the conversion of heat into mechanical work and vice versa is studied.

Thermodynamical System & Important terms

The system which can be represented in of pressure (P), volume (V) and temperature (T), is known thermodynamic system. A specified portion of matter consisting of one or more substances on which the effects of thermodynamic variables such as temperature, volume and pressure are to be studied, is called a system. e.g. A gas enclosed in a cylinder fitted with a piston is a system.

Heterogeneous System

A system which is not uniform throughout is said to be heterogeneous. e.g. A system consisting of two or more immiscible liquids.

Homogeneous System

A system is said to be homogeneous if it is completely uniform throughout. e.g. Pure solid or liquid.

Surroundings

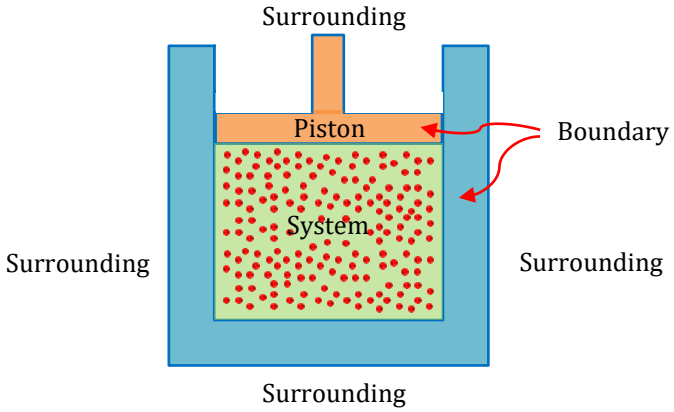
Anything outside the system, which exchanges energy with the system and which tends to change the properties of the system is called its surroundings.

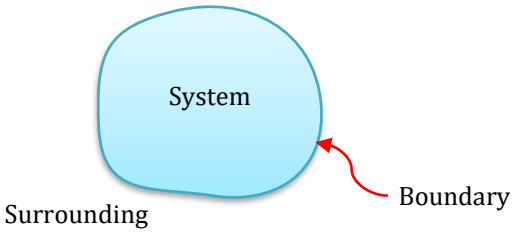
Universe

The system and its surroundings are together known as the universe.

Boundary

Real or imaginary wall that separates the system from the surroundings





Types of Systems

(1) **Open System :** Any interaction is possible.

(2) **Closed System :** Impenetrable by matter, but other kinds of interactions can occur.

(3) **Semipermeable :** Penetrable by some species, but not by others; rest interactions possible.

(4) **Insulated System :** Thermal interactions are not possible, but nonthermal interactions can occur.

(5) **Rigid System :** Boundary cannot be mechanically deformed.

(6) **Isolated System :** No interactions can occur.

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Closed system

Open system

Isolated System

- **Intensive**
  Independent of the size (or mass) of the system. Example:- pressure, temperature etc.
- **Extensive**
  depends on the size (or mass) of the system. Example:- volume, number of moles etc.

Thermodynamic variables of the system :

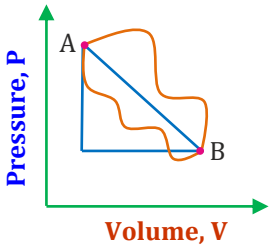
(i) Composition (m) (ii) Temperature (T) (iii) Volume (V)
(iv) Pressure (P) (v) Mass

Thermodynamic state

The state of a system can be described completely by composition, temperature, volume and pressure.
If a system is homogeneous and has definite mass and composition, then the state of the system can be described by the remaining three variables namely temperature, pressure and volume. These variables are interrelated by equation PV = μRT. The thermodynamic state of the system is its condition as identified by two independent thermodynamic variables (P, V or P, T or V, T).

Process

When a system changes from one equilibrium state to another the path of successive states through which the system passes is called a process.



- **Cyclic process**
  Cyclic process is a thermodynamic process in which the system returns to its initial stage after undergoing a series of changes.
- **Non–cyclic process**
  Non–cyclic process is a process in which the system does not return to its initial stage.
- **Quasi–static or equilibrium process**
  Quasi–static is a thermodynamic process which proceeds extremely slowly such that at every instant of time, the temperature and pressure are the same in all parts of the system.
- **Reversible and Irreversible processes**
  A reversible process is one in which the changes in heat and work of direct process from initial to a final state are exactly retraced in opposite sense in the reverse process and the system and surroundings are left in their initial states. The reversibility is an ideal concept and can not be realized in practice.
  The process which is not reversible is the irreversible process. In nature the processes are irreversible.

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Heat and Work done in Thermodynamic Process

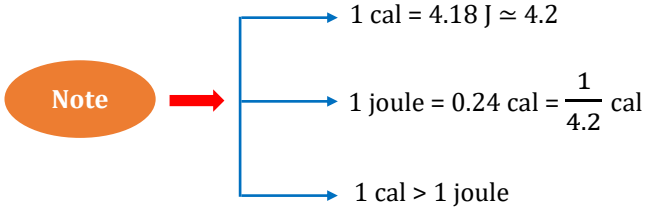
Work and Heat:

$W \propto Q$ (Heat)

$W = JQ$

Here J = conversion factor = Joule's constant (mechanical equivalent of heat)

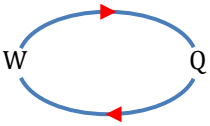
$J = 4.18 \frac{\text{joule}}{\text{calorie}}$ (Dimensionless)



Special Point :

(a) Work and heat both are the form of energy. Work is ordered energy where as heat is disordered form of energy.

(b) Conversion of mechanical work into heat is completely possible but its reverse is not true.



100% ✓

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Two Types of Conversions:

(a) Mechanical work $\rightarrow$ Heat

$$\text{PE}: \frac{mgh}{J} = msd\theta(\text{cal})\left\{\frac{1}{J} = 0.24\right\}$$

$$\text{KE}: \frac{1}{2}\frac{mv^2}{J} = msd\theta(\text{cal})$$

$$\text{Rotational kinetic energy} = \frac{1}{2}\frac{I\omega^2}{J} = msd\theta(\text{cal})$$

(b) Heat $\rightarrow$ mechanical work

Illustration 94:

Calculate rise in temperature of water when falls from 100m high waterfall.

Solution:

Conversion PE $\rightarrow$ heat

$$\frac{mgh}{J} = msd\theta \qquad \Rightarrow \qquad d\theta = \frac{(10)(100)}{4.2\times1000} = 0.24^\circ C$$

Illustration 95:

A block of 100 gm is sliding on a rough horizontal surface. If speed of block reduces from 10 m/sec to 5 m/sec then find heat generated in this process.

Solution:

Heat generated = change in KE

$$\Rightarrow \quad \text{heat generated} = \frac{1}{2}m\left[v_1^2 - v_2^2\right] = \frac{1}{2}\times100\times10^{-3}\left[(10)^2 - (5)^2\right] = \frac{75}{2}\times10^{-1} = 3.75 \text{ joule}$$

Illustration 96:

A bullet of mass 84 gm is moving with a velocity of 200 m/s. It collides with wall by which one fourth of the kinetic energy is converted into heat. Find the heat developed by bullet in calorie.

Solution:

$$Q = \frac{1}{4}(W) = \frac{1}{4}\left(\frac{1}{2}\frac{mv^2}{J}\right) = \frac{1}{4}\times\frac{1}{2}\times\frac{84}{1000}\times\frac{200\times200}{4.2} = 100 \text{cal.}$$

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Illustration 97:
A man gains 100 kcal of heat by eating bread. His efficiency is 25% and mass is 70 kg. Calculate the height to which he can climb.

Solution:
$$\frac{25}{100}\times Q = W$$
$$\frac{25}{100}\times 100\times 10^3 = \frac{Mgh}{J}$$
$$25\times 10^3 = \frac{70\times 10\times h}{4.2}$$
h = 150 meter

Internal Energy
Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p).

$dU = dU_k + dU_p$

If there no intermolecular forces, then $dU_p = 0$ and $dU = dU_k = m\, c_v\, dT$ [ideal gas]

c_v = Specific heat at constant volume and dT = Infinitesimal change in temperature

m = Mass of system M = Molecular weight

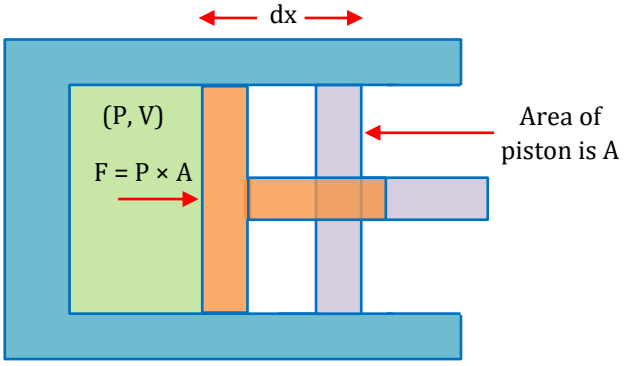
Molar heat capacity $C_v = Mc_v$ For μ–moles of ideal gas = $dU = \mu C_v dT = \frac{m}{M} C_v dT$

Internal energy in the absence of inter–molecular forces is simply the function of temperature and state only, it is independent of path followed. $\Delta U = U_f - U_i$

U_i = Internal energies in initial state and U_f = Internal energies in final state

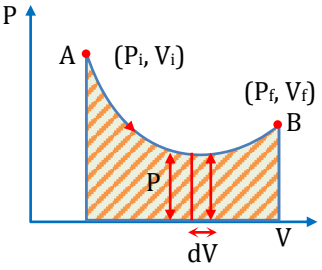
Properties of Internal Energy
1. It depends mainly on states of matter and temperature of system.
2. For ideal gas it a function of temperature, identity and moles.
3. For real gas it is a function of T and V or T and P as well as identity and moles.
4. Absolute value of internal energy is impossible to determine, so it is generally expressed as a difference between the two states of system, or by applying zero value to a reference state.
5. Internal energy is an Exact quantity or a state function.
6. Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.
7. The energy due to molecular motion is called internal kinetic energy (U_k) and that due to molecular configuration is called internal potential energy (U_p).

Work done by thermodynamic system
One of the simple example of a thermodynamic system is a gas in a cylinder with a movable piston.



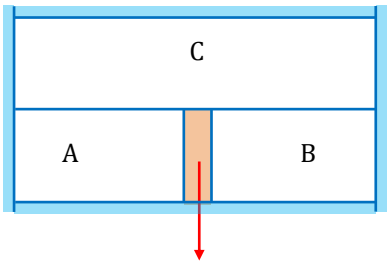
- If the gas expands against the piston Gas exerts a force on the piston and displace it through a distance and does work on the piston.

- If the piston compresses the gas When piston moved inward, work is done on the gas.
- The work associated with volume changes
  If pressure of gas on the piston = P.
  Then the force on the piston due to gas is F = PA
- When the piston is pushed outward an infinitesimal distance dx, the work done by the gas is dW = F × dx = PA dx
  The change in volume of the gas is dV = Adx, ∴ dW = PdV
  For a finite change in volume from Vi to Vf, this equation is then integrated between V_i to V_f to find the net work done = $W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} PdV$
  Hence the work done by a gas is equal to the area under P–V graph.



Zeroth and First Law of Thermodynamics

If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduce the concept of temperature. Two objects (or systems) are said to be in thermal equilibrium if their temperatures are the same.



- If two systems are separately in a thermal equilibrium with the third system then they must be in thermal equilibrium with each other.
- ZLOT defines temperature only.
- In measuring the temperature of a body, it is important that the thermometer be in the thermal equilibrium with the body whose temperature is to be measured.

First law of thermodynamics

If some quantity of heat is supplied to a system capable of doing external work, then the quantity of heat absorbed by the system is equal to the sum of the increase in the internal energy of the system and the external work done by the system.

$$\boxed{\delta Q = dU + \delta W} \quad \text{or} \quad \boxed{Q = W + \Delta U}$$

- This law is applicable to every process in nature.
- Thermodynamic force is non-conservative force.
- The first law of thermodynamics introduces the concept of internal energy.
- The first law of thermodynamics is based on the law of conservation of energy.
- δQ, dU and δW must be expressed in the same units (either in units of work or in units of heat).
- This law is applicable to all the three phases of matter, i.e., solid, liquid and gas.
- dU is a characteristic of the state of a system, it may be any type of internal energy–translational kinetic energy, vibrational, rotational kinetic energy, binding energy etc.

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Sign Convention used in Physics Thermodynamics

FLOT $\Delta Q = dU + \Delta W$ or $[Q = \Delta U + W]$

Rules:

1. $Q/\Delta Q =$
   - +ve $\Rightarrow$ heat given to system / heat absorbed by system.
   - –ve $\Rightarrow$ heat rejected / released by system or heat taken out from system
2. $\Delta U/dU =$
   - +ve $\Rightarrow$ U ↑ (internal energy) T ↑ (increases)
   - –ve $\Rightarrow$ U ↓ (internal energy) T ↓ (decreases)
3. $W/\Delta W =$
   - V ↑ (expansion) dV = +ve $\Rightarrow$ $\Delta W = PdV$ = +ve (work is done by system)
   - V ↓ (compression) ; dV = –ve $\Rightarrow$ $\Delta W = PdV$ = –ve (work is done on system)

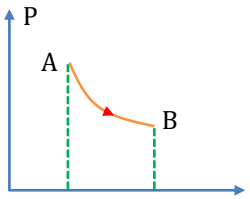
Work Done in Various Thermodynamic Process:

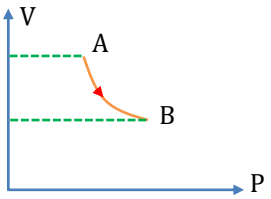
(1) Mathematical method $W = \int_{V_i}^{V_f} PdV$

Process	W
Cyclic	Area of the closed curve
Isochoric	0
Isothermal	$\mu RT\log_e\left[\frac{V_f}{V_i}\right] = \mu RT\log_e\left[\frac{P_i}{P_f}\right]$
Adiabatic	$\frac{\mu R(T_f - T_i)}{1-\gamma}$
Isobaric	$P(V_f - V_i) = \mu R(T_f - T_i)$
Polytropic process	$\frac{\mu R(T_f - T_i)}{1-x}$

(2) **Graphical method ΔW = Area enclosed between PV curve and Volume axis.**

Example: -





Work done $\Rightarrow W_{A\to B}$ = Area shaded.

Sign concept: -

- Expansion work → +ve (work by system)/volume of a gas increases
- Compression work → –ve (work on system)/ volume of a gas decreases

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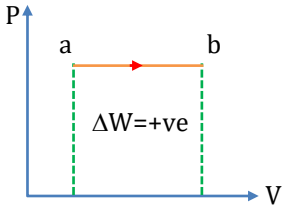
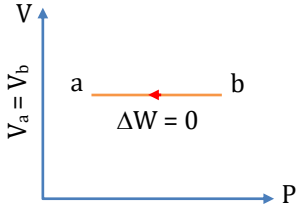
(i)



(ii)



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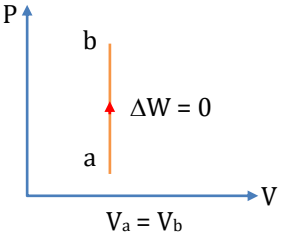

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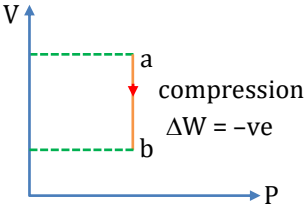

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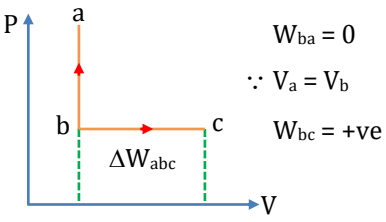
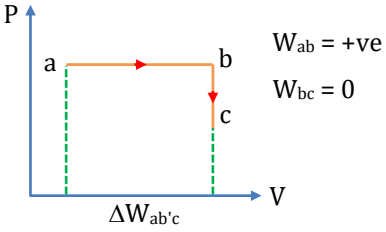


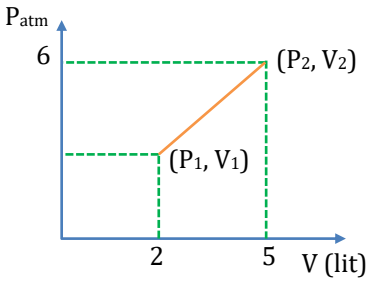
(vi)



Note: Area enclosed between PV curve and volume axis.

Illustration 98:

A system changes from the state (P_1,V_1) to (P_2,V_2) as shown in figure. What is the work done by the system?



Solution:

W = Area between curve line and volume axis.

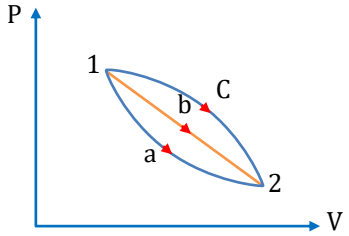
$$W = \frac{1}{2}\times\left[(2+6)\times10^5\right]\times\left(3\times10^{-3}\right)$$

W = 1200 joule

Note: (a) work and heat are both path functions, it depends on path

(b) internal energy depends on initial and final state, it is a state function.

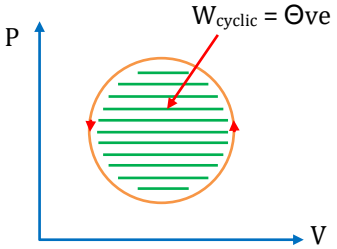
- **Order of work :**
  $\Delta W_C > \Delta W_b > \Delta W_a$
- **Order of internal energy:**
  $dU_a = dU_b = dU_c$ (constant)
- **Order of heat:**
  $\Delta Q_c > \Delta Q_b > \Delta Q_a$
  $\therefore$ $Q = W + \Delta U$



Cyclic Process:

1. Work done in cyclic process :

(a) ΔW = Area enclosed between closed loop



(b) In P-V graph

Direction → C.W. $\Rightarrow \Delta W = +ve$

Direction → ACW $\Rightarrow \Delta W = -ve$

(c) In V-P graph

Direction → C.W. : $\Delta W = -ve$

Direction → ACW $\Rightarrow \Delta W = +ve$

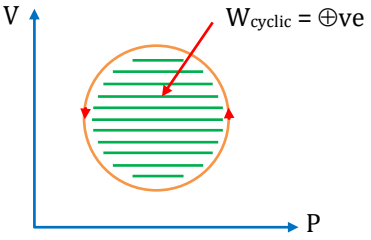
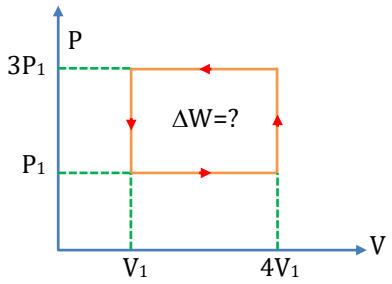


Illustration 99:



Solution:

1. Area = $2P_1 \times 3V_1 = 6P_1V_1$
   Work = $-6P_1V_1$ Joule [$\because$ Cycle is ACW]
2. In cyclic process: dU = 0
3. Heat: $\Delta Q = \Delta W$ [By FLOT $\Delta Q = dU + \Delta W$]

Illustration 100:

In a thermodynamic process, work done on the system is 100 J and heat given to system is 500 cal. Then calculate change in internal energy of system.

Solution:

As from FLOT : $\Delta Q = dU + \Delta W$

$\Rightarrow$ $(500 \times 4.2) = dU - 100$

dU = 2100 + 100 = 2200 J = 524 cal dU is positive so temperature will increase

Illustration 101:

During an experiment heat released by the system is 10 cal and work done on the system is 10 J. If initial internal energy of the system is 50 J then find final internal energy.

Solution:

As $\Delta Q = \Delta W + U_f - U_i$

-10 cal $= -10$ J $+ U_f - 50$

$\Rightarrow$ $U_f = -10 \times 4.2 + 10 + 50$

$\Rightarrow$ $U_f = -42 + 60 = 18$ J

Illustration 102:

A gas under constant pressure of 4.5×10^5 Pa when subjected to 800 kJ of heat, changes the volume from $0.5m^3$ to 2.0 m^3. Find the change in internal energy of the gas?

Solution:

$\Delta U = \Delta Q - \Delta W = \Delta Q - P\Delta V$

$\Delta U = 800 \times 10^3 - 4.5 \times 10^5 (2 - 0.5) = 1000(800 - 675)$

$\Rightarrow dU = 125$ kJ

Illustration 103:

If Q amount of heat is given to a diatomic ideal gas in process in which the gas perform a work $\frac{2Q}{3}$ its surrounding. Find the molar heat capacity (in terms of R) for the process.

Solution:

According to FLOT

$Q = W + \Delta U$

$$Q = \frac{2}{3}Q + \mu C_v \Delta T$$

$$\frac{Q}{3} = \mu \times \frac{5}{2} R\Delta T$$

$$\mu C\Delta T = \frac{15}{2}\mu R\Delta T \Rightarrow C = \frac{15}{2}R$$

Illustration 104:

The pressure of one mole monoatomic gas increases linearly from 4×10^5 Nm^{-2} to $8 \times 10^{+5}$ Nm^{-2} when its volume increases from $0.2m^3$ to 0.5 m^3. Calculate.

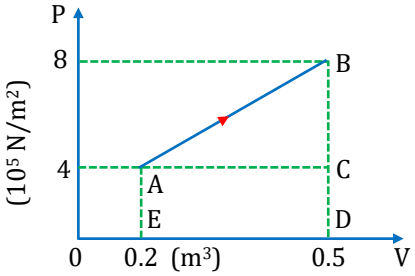
(i) Work done by the gas, (ii) Increase in the internal energy,

(iii) Amount of heat supplied, (iv) Molar heat capacity of the gas R = 8.31 J mol^{-1} K^{-1}

Solution:

$P_1 = 4 \times 10^5$ Nm^{-2}

$P_2 = 8 \times 10^{+5}$ Nm^{-2}, $V_1 = 0.2$ m^3, $V_2 = 0.5$ m^3



(i) Work done by the gas = Area under P–V graph (Area ABCDEA)

$$(AE + BD) \times AC = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5 - 0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$

(ii) Increase in internal energy $\Delta U = C_V (T_2 - T_1) = \frac{C_V}{R} R(T_2 - T_1)$

$$= \frac{C_V}{R}(P_2V_2 - P_1V_1)$$

For monoatomic gas $C_V = \frac{3}{2}R$

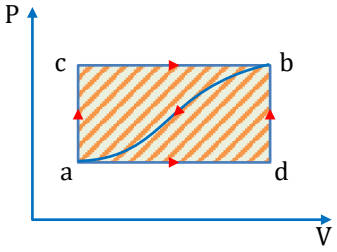
$$\therefore \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

(iii) $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5$ J

(iv) $C = \frac{Q}{\mu \Delta T} = \frac{QR}{\mu R \Delta T} = \frac{QR}{(P_2V_2 - P_1V_1)} = \frac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5}$ 17.14 J/mol K

Illustration 105:

As shown in figure when a system is taken from state a to state b, along the path



$a \rightarrow c \rightarrow b$, 60 J of heat flow into the system, and 30 J of work is done:

(i) How much heat flows into the system along the path $a \rightarrow d \rightarrow b$ if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process $a \rightarrow d$ and $d \rightarrow b$.

Solution:

For the path acb $\Delta U = Q - W = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path adb $Q = \Delta U + W = 30 + 10 = 40$ J

(ii) Along the curved path ba $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ J, heat liberates from system

(iii) $Q_{ad} = U_d - U_a + W_{ad}$

but $W_{ad} = W_{adb} - W_{db} = 10 - 0 = 10$ Hence $Q_{ad} = 22 - 0 + 10 = 32$ J

and $Q_{db} = U_b - U_d + W_{db} = 30 - 22 + 0 = 8$ J

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Illustration 106:

1 kg of water at 373 K is converted into steam at same temperature. Volume of 1 cm^3 of water becomes 1671 cm^3 on boiling. What is the change in the internal energy of the system if the latent heat of vaporisation of water is 5.4×10^5 cal kg^{-1}?

Solution:

Volume of 1 kg of water = 1000 $cm^3 = 10^{-3}$ m^3, Volume of 1 kg of steam = $10^3 \times 1671$ $cm^3 = 1.671$ m^3 Change in volume $\Delta V = (1.671 - 10^{-3})$ $m^3 = 1.670$ m^3, Pressure, P = 1 atm. = 1.01×10^5 N m^{-2} In expansion work done,

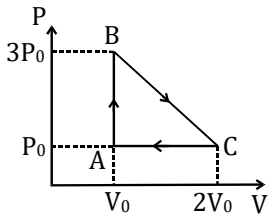
$W = P\Delta V = 1.01 \times 10^5 \times 1.67$ J = $\frac{1.686 \times 15^5}{4.2}$ cal = 4.015×10^4 cal

But $\Delta U = Q - W$ (first law of thermodynamics)

or $\Delta U = (5.4 \times 10^5 - 0.4015 \times 10^5)$ cal = 4.9985×10^5 cal

BEGINNER'S BOX-7

1. One mole of an ideal monoatomic gas is taken round the cyclic process ABCA as shown in fig. calculate.

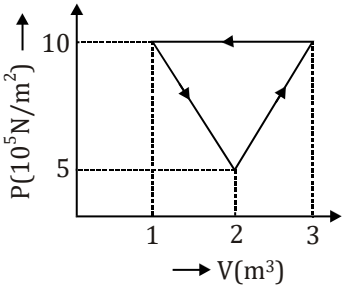


(a) The work done by the gas.

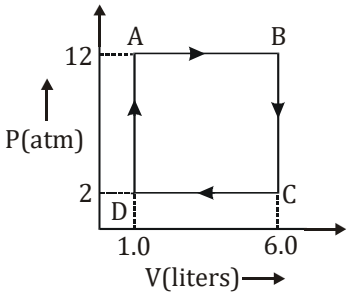
(b) The heat rejected by the gas in the path CA and heat absorbed in the path AB.

(c) The net heat absorbed by the gas in the path BC.

2. Calculate the heat supplied in figure shown for complete cycle.



3. Given figure shows a P–V graph of the thermodynamic behaviour of an ideal gas. Find out work in the processes A → B, B → C, C → D and D → A from this graph.



4. At 8 atm pressure a monoatomic gas expands from 2 litre to 5 litre then calculate the following

(a) Work done by the gas

(b) Increase in internal energy

(c) Amount of heat supplied

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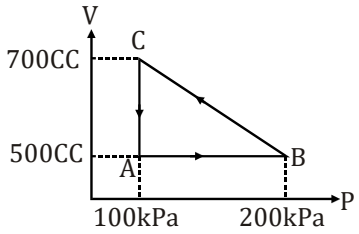
5. A gas is taken through a cyclic process ABCA as shown in figure. If 2.4 cal of heat is given in the process. Calculate the value of J.



Different Thermodynamics Process

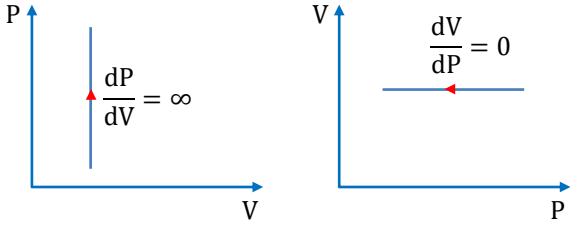
Isometric or Isochoric Process

Isochoric process is a thermodynamic process that takes place at constant volume of the system, but pressure and temperature varies for change in state of the system.

Equation of state P/T = constant (P and T are variable, V is constant)

Work done In this process volume remains constant so dV = 0 $\Rightarrow W = \int_{V_i}^{V_f} PdV = 0$

Form of first Law Q = ΔU



It means whole of the heat supplied is utilized for change in internal energy of the system. $Q = \Delta U = \mu C_v DT$

Slope of the P–V curve $\frac{dP}{dV} = \infty$

Specific heat or molar heat capacity at constant volume (C_V)

The quantity of heat required to raise the temperature of 1-gram mole of gas through 1 °C at constant volume is equal to the specific heat at constant volume.

- A gas enclosed in a cylinder having rigid walls and a fixed piston. When heat is added to the gas, there would be no change in the volume of the gas.
- When a substance melts, the change in volume is negligibly small. So, this may be regarded as a nearly isochoric process.
- Heating process in pressure cooker is an example of isometric process.

Isobaric Process

Isobaric process is a thermodynamic process that takes place at constant pressure, but volume and temperature varies for change in state of the system.

- **Equation of state** V/T = Constant or $V \propto T$
- **Work done** In this process pressure remains constant $\therefore$ dP = 0

  Work done $W = \int_{V_i}^{V_f} PdV = P(V_f - V_i)$

$\Rightarrow$ Fraction of Heat given which –

(i) is converted into internal energy $\boxed{\frac{\Delta U}{\Delta Q} = \frac{1}{\gamma}}$

(ii) does work against external pressure $\boxed{\frac{\Delta W}{\Delta Q} = 1 - \frac{1}{\gamma}}$

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- **Form of first Law** $Q = \Delta U + P(V_f - V_i)$

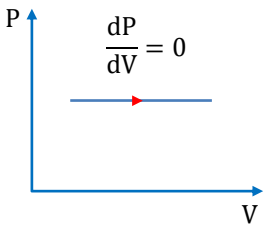
  $C_p(T_f - T_i) = \mu C_v(T_f - T_i) + P(V_f - V_i)$

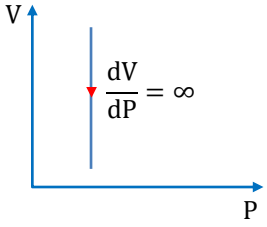
  It is clear that heat supplied to the system is utilized for:

(i) Increasing internal energy and

(ii) Work done against the surrounding atmosphere.

- **Slope of the PV curve:** $\left(\frac{dP}{dV}\right)_{isobaric} = 0$





Isothermal Process

In this process pressure and volume of system change but temperature remains constant. In an isothermal process, the exchange of heat between the system and the surroundings is allowed. Isothermal process is carried out by either supplying heat to the substance or by extracting heat from it. A process has to be extremely slow to be isothermal.

- **Equation of state:** P V = constant (μRT) (T is constant)
- **Work Done**

  Consider μ moles of an ideal gas, enclosed in a cylinder, at absolute temperature T, fitted with a frictionless piston. Suppose that gas undergoes an isothermal expansion from the initial state (P_1, V_1) to the final state

  (P_2, V_2) Work done: $W = \int_{V_1}^{V_2} PdV \quad \therefore PV = \mu RT$ then $P = \frac{\mu RT}{V}$

  $$\therefore W = \int_{V_1}^{V_2} \frac{\mu RT}{V} dV = \mu RT \int_{V_1}^{V_2} \frac{dV}{V} = \mu RT \left[\log_e V\right]_{V_1}^{V_2} = \mu RT\left[\log_e V_2 - \log_e V_1\right] = \mu RT \log_e \left[\frac{V_2}{V_1}\right]$$

  $$\Rightarrow W = 2.303\ \mu RT \log_{10}\left[\frac{P_1}{P_2}\right] \qquad [\because P_1V_1 = P_2V_2]$$

- **Form of First Law**

  There is no change in temperature and internal energy of the system depends on temperature only

  So $\Delta U = 0, Q = 2.303\ \mu RT \log_{10}\left[\frac{V_2}{V_1}\right]$

  It is clear that Whole of the heat energy supplied to the system is utilized by the system in doing external work. There is no change in the internal energy of the system.

- **Slope of the isothermal curve**

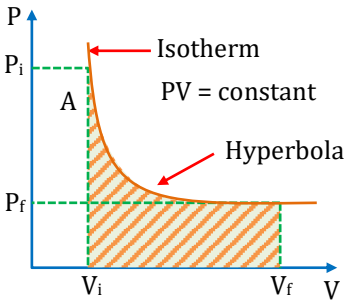
  For an isothermal process, PV = constant

  Differentiating, PdV + VdP = 0

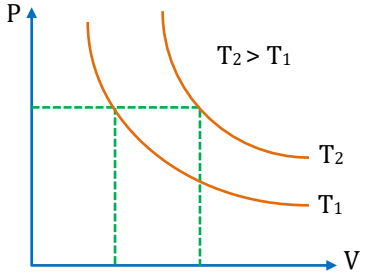
  $\Rightarrow VdP = -PdV$

  $\Rightarrow \frac{dP}{dV} = -\frac{P}{V}$

  Slope of isothermal curve, $\left[\frac{dP}{dV}\right]_{isothermal} = -\frac{P}{V}$



- **For a given system :**
- The product of the pressure and volume of a given mass of a perfect gas remains constant in an isothermal process.
- Boyle's law is obeyed in an isothermal process.
- A graph between pressure and volume of a given mass of a gas at constant temperature is known as isotherm or isothermal curve of the gas.
- Two isotherms for a given gas at two different temperatures T_1 and T_2 are shown in figure.



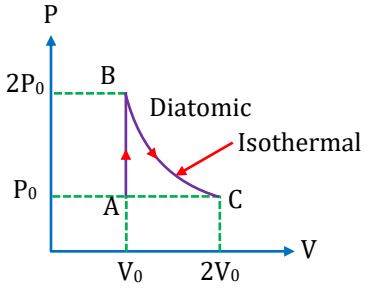
At const P ; $V \propto T$ so $T_2 > T_1$

- The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.
- If two isotherms intersect each other at a single point we get same value of P and V at intersection point. $PV = \mu RT_1$ for temperature T_1 and $PV = \mu RT_2$ for temperature T_2.
  It means $T_1 = T_2$ which is not possible.
- An ideal gas enclosed in a conducting cylinder fitted with a conducting piston.
- Let the gas be allowed to expand very–very slowly. This shall cause a very slow cooling of the gas, but heat will be conducted into the cylinder from the surrounding. Hence the temperature of the gas remains constant. If the gas is compressed very–very slowly, heat will be produced, but this heat will be conducted to the surroundings and the temperature of the gas shall remain constant.
- The temperature of a substance remains constant during melting. So, the melting process is an isothermal process.
- Boiling is an isothermal process, when a liquid boils, its temperature remains constant.
- If sudden changes are executed in a vessel of infinite conductivity then they will be isothermal.

Illustration 107:

P-V curve of a diatomic gas is shown in the figure. Find the total heat given to the gas in the process AB and BC

Solution:



From first law of thermodynamics

$\Delta Q_{ABC} = \Delta U_{ABC} + W_{ABC}$

$W_{ABC} = W_{AB} + W_{BC} = 0 + nRT_B \ln \frac{V_C}{V_B} = nRT_B \ln \frac{2V_0}{V_0}$

$= nRT_B \ln 2 = 2P_0V_0 \ln 2$

$\Delta U = nC_V\Delta T = \frac{5}{2}(2P_0V_0 - P_0V_0)$

$\Rightarrow \Delta Q_{ABC} = \frac{5}{2} P_0V_0 + 2P_0V_0 \ln 2$

Adiabatic Process

It is that thermodynamic process in which pressure, volume and temperature of the system do change but there is no exchange of heat between the system and the surroundings.

A sudden and quick process will be adiabatic since there is no sufficient time available for exchange of heat so process adiabatic.

Equation of state : $PV = \mu RT$

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Equation for adiabatic process PV^{γ} = constant, $TV^{\gamma-1}$ = constant, $T^{\gamma}p^{1-\gamma}$ = constant

Work done

Let initial state of system is (P_1, V_1, T_1) and after adiabatic change final state of system is (P_2, V_2, T_2) then we can write $P_1V_1^{\gamma} = P_2V_2^{\gamma} = K$ (here K is const.)

So $$W = \int_{V_1}^{V_2} PdV = K\int_{V_1}^{V_2} V^{-\gamma}dV = K\left(\frac{V^{-\gamma+1}}{-\gamma+1}\right)_{V_1}^{V_2} = \frac{K}{(-\gamma+1)}\left[V_2^{-\gamma+1} - V_1^{-\gamma+1}\right] \quad \left(\because\ K = P_1V_1^{\gamma} = P_2V_2^{\gamma}\right)$$

$$\Rightarrow W = \frac{1}{(\gamma-1)}\left[P_1V_1^{\gamma}V_1^{-\gamma}.V_1 - P_2V_2^{\gamma}V_2^{-\gamma}.V_2\right] = \frac{1}{(\gamma-1)}\left[P_1V_1 - P_2V_2\right] \Rightarrow W = \frac{\mu R}{(\gamma-1)}\left(T_1 - T_2\right) \quad \left(\because PV = \mu RT\right)$$

Form of first law : $dU = -\delta W$

It means the work done by an ideal gas during adiabatic expansion (or compression) is on the cost of change in internal energy proportional to the fall (or rise) in the temperature of the gas.

If the gas expands adiabatically, work is done by the gas. So, W_{adia} is positive.

The gas cools during adiabatic expansion and $T_1 > T_2$.

If the gas is compressed adiabatically, work is done on the gas. So, W_{adia} is negative.

The gas heats up during adiabatic compression and $T_1 < T_2$.

Slope of the adiabatic curve

For an adiabatic process, PV^{γ} = constant

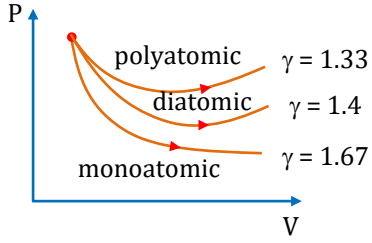
Differentiating, $\gamma PV^{\gamma-1}dV + V^{\gamma}dP = 0$

$\Rightarrow$ $V^{\gamma}dP = -\gamma PV^{\gamma-1}dV$

$\Rightarrow$ $$\frac{dP}{dV} = -\frac{\gamma PV^{\gamma-1}}{V^{\gamma}} = -\gamma\frac{P}{V} = \gamma\left(-\frac{P}{V}\right)$$

Slope of adiabatic curve, $\left[\frac{dP}{dV}\right]_{adiabatic} = -\frac{\gamma P}{V}$

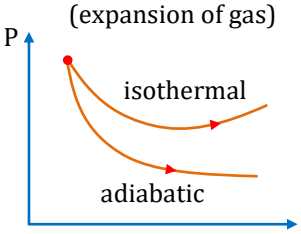
(Adiabatic expansion of gas)



Magnitude of slope of adiabatic curve is greater than the slope of isotherm

$$\left|\frac{dP}{dV}\right|_{adiabatic} = \gamma\frac{P}{V} = \gamma\left|\frac{dP}{dV}\right|_{isothermal} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$$

(expansion of gas)



Since γ is always greater than one so an adiabatic curve is steeper than an isothermal curve

Examples of adiabatic process

- A gas enclosed in a thermally insulated cylinder fitted with a non–conducting piston. If the gas is compressed suddenly by moving the piston downwards, some heat is produced. This heat cannot escape from the cylinder. Consequently, there will be an increase in the temperature of the gas.
- If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.
- Bursting of a cycle tube.
- Propagation of sound waves in a gaseous medium.
- In diesel engines burning of diesel without spark plug is done due to adiabatic compression of diesel vapour and air mixture

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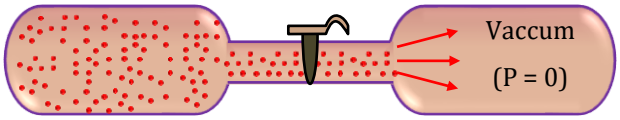
Free Expansion

Take a thermally insulated bottle with ideal gas at some temperature T_1 and, by means of a pipe with a stopcock, connect this to another insulated bottle which is evacuated. If we suddenly open the stopcock, the gas will rush from the first bottle into the second until the pressures are equalized.



Free expansion of a gas

Experimentally, we find that this process of free expansion does not change the temperature of the gas – when the gas attains equilibrium and stops flowing, the final temperature of both bottles are equal to the initial temperature T_1. This process is called a free expansion.

The change in the internal energy of the gas can be calculated by applying the first law of thermodynamics to the free–expansion process.

The process is adiabatic because of the insulation, So Q = 0.

No part of the surroundings moves so the system does no work on its surroundings.

For ideal gas $(\gamma W)_{ext.}$ = Work done against external atmosphere = PdV = 0

$(\because P = 0) \Rightarrow$ because vacuum

$(\delta W)_{int.}$= Work done against internal molecular forces = 0

$\delta Q = dU + \delta W \Rightarrow 0 = dU + 0$

The internal energy does not change dU = 0 So U and T are constant.

The initial and final states of this gas have the same internal energy.

Which implies that the internal energy of an ideal gas does not depend on the volume at all.

The free–expansion process has led us to the following conclusion:

The internal energy U(T) of an ideal gas depends only on the temperature.

Illustration 108:

Temperature and pressure of air in tyre tube is 27°C and 8 atm respectively. If tube suddenly burst then calculate final temperature of air. [γ = 1.5)

Solution:

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma} \Rightarrow (8)^{1-\gamma}(300)^{\gamma} = (1)^{1-\gamma}(T_2)\gamma \Rightarrow T_2 = 150\ K \Rightarrow T_2 = 150 - 273 = -123°C$$

Illustration 109:

A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_1K and final temperature is aT_1 then find the value of a.

Solution:

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow T_1(V)^{\frac{7}{5}-1} = a.T_1\left(\frac{V}{32}\right)^{\frac{7}{5}-1} \Rightarrow a = 2^{5\times\frac{2}{5}} = 4$$

Illustration 110:

Find the order of magnitude of slope of PV curve in different process.

Solution:

Slope IC > AD > IT > IB

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\infty \quad \frac{\gamma P}{V} \quad \frac{P}{V} \quad 0$$

IC [Isochoric process] or IM [Iso volumetric process] or Isometric process

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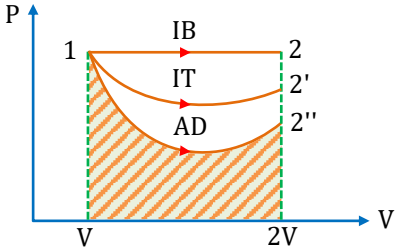
Illustration 111:

A gas is expanded to double of its initial volume through three different process (IB, IT, AD) then find



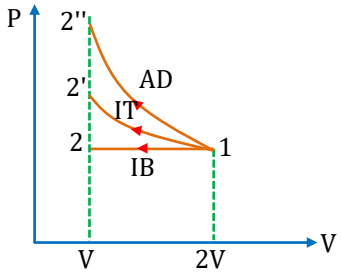
(a) Order of work done (b) Order of final pressure (c) Order of final temperature

Solution:

(a) IB > IT > AD (b) IB > IT > AD (c) IB > IT > AD

Illustration 112:

A gas is compressed to half of its initial volume along three different process, then



(a) order of work done (b) Order of final pressure (c) Order of temperature

Solution:

(a) AD> IT > IB (b) AD > IT > IB (c) AD > IT > IB

Illustration 113:

An ideal gas $\left(\gamma=\frac{5}{3}\right)$ is adiabatically compressed from to 80 cm3 to 10 cm3. If the initial pressure is P then find the final pressure?

Solution:

$$\therefore\ P_1V_1^{\gamma}=P_2V_2^{\gamma}\Rightarrow\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^{\gamma}\Rightarrow\frac{P}{P_2}=\left(\frac{10}{80}\right)^{\frac{5}{3}}\Rightarrow P_2=32P$$

Polytropic Process

General expression for C (C_P or C_V) in the process PV^x = constant $C=\frac{R}{\gamma-1}+\frac{R}{\gamma-1}$

For isobaric process P = constant so x = 0 $\therefore C=C_P=\frac{R}{\gamma-1}+R=C_V+R$

For isothermal process, PV = constant so x = 1 $\therefore$ C = ∞

For adiabatic process PV^{γ} = constant so x = g $\therefore$ C = 0

Values of f, U, C_V, C_P and γ for different gases are shown in table below.

Atomicity of gas	f	C_v	C_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3}=1.67$
Diatomic, Triatomic and Triatomic linear (at normal temperature)	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5}=1.4$
Poly atomic Triangular Non-linear	6	$\frac{6}{2}R=3R$	$\frac{8}{2}R=4R$	$\frac{4}{3}=1.33$

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Illustration 114:

Find the molar heat capacity (in terms of R) of a monoatomic ideal gas undergoing the process:

$PV^{1/2}$ = constant?

Solution:

$PV^{1/2} = K \qquad \Rightarrow V = \left(\frac{K}{P}\right)^2$

$P = \frac{\mu RT}{K^2} \times P^2 \qquad \therefore P = \frac{K^2}{\mu RT}$

$As\ V^{12} = \frac{K}{P} = \frac{\mu RT}{K} \qquad \therefore dv = \frac{\mu^2 R^2 2T \cdot dt}{K^2}$

$dW = P \cdot dv = \frac{K^2}{\mu RT} \times \frac{\mu^2 R^2 \cdot 2T \cdot dT}{K^2}$

$dW = 2\mu RdT$

Now, dQ = dU + dW

$\mu CdT = \frac{\mu 3RdT}{2} + 2\mu RdT \qquad \therefore C = \frac{7}{2}R$

Alter: -

Use polytropic process

$C = C_v + \frac{R}{1-x}$

PV^x = constant

Illustration 115:

An ideal gas is taken through a process in which pressure and volume vary as $P = kV^2$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + \frac{R}{3}$

Solution:

$PV^{-2} = K$

$As\ C = C_v + \frac{R}{(1-x)}, x = -2 = C_V + \frac{R}{1-(-2)} \Rightarrow C = C_v + \frac{R}{3}.$

Illustration 116:

A monoatomic ideal gas ($C_p/C_v = \gamma$) is taken through a process in which the pressure and the volume vary as $P = kV^a$, where k = constant. Find the value of a for which the molar specific heat capacity in the process is zero.

Solution:

$C = C_v + \frac{R}{1-x}$; If PV^x = constant

$0 = \frac{3}{2}R + \frac{R}{1+a}; a = -\frac{5}{3}$

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Illustration 117:

Match the column-I with column-II

	Column-I		Column-II
(A)	Adiabatic expansion	(p)	Q = 0
(B)	Adiabatic free expansion	(q)	$\Delta U = 0$
(C)	Isobaric expansion	(r)	W = 0
(D)	Isothermal expansion	(s)	$Q = W + \Delta U$
(E)	Isochoric cooling		

Solution:

(A) – p,s, (B) – p,q,r,s, (C) – s , (D) – q,s, (E) – r,s

BEGINNER'S BOX-8

1. 32 gram of oxygen gas at temperature 27°C is compressed adiabatically to 1/3 of its initial volume. Calculate the change in internal energy? (γ = 1.5 for oxygen)

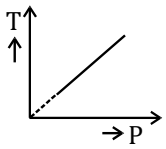
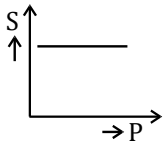
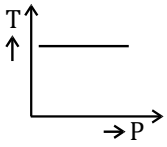
2. How much heat energy should be added to the gaseous mixture consisting of one gram of hydrogen and one gram helium to raise its temperature from 0°C to 50°C.

(a) at constant volume and

(b) at constant pressure, assuming both the gases to be ideal.

R = 2 cal/mol-K, $\gamma_{H_2} = 1.41$, $\gamma_{He} = 1.67$

3. **Match the column–I with column–II**

Column – I	Column – II
(A) 	(p) Isometric process
(B) 	(q) Adiabatic process
(C) 	(r) Isothermal process

4. **Match the column–I with column–II**

Column–I		Column–II	
(A)	Isobaric process	(p)	No heat exchange
(B)	Isothermal process	(q)	Constant pressure
(C)	Isoentropic process	(r)	Constant internal energy
(D)	Isochoric process	(s)	Work done is zero

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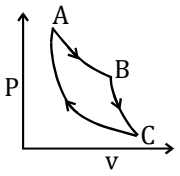
5. **Match the column–I with column–II**

Column–I		Column–II	
(A)	Adiabatic expansion	(p)	Q = 0
(B)	Adiabatic free expansion	(q)	$\Delta U = 0$
(C)	Isobaric expansion	(r)	W = 0
(D)	Isothermal expansion	(s)	$Q = W + \Delta U$
(E)	Isochoric cooling		

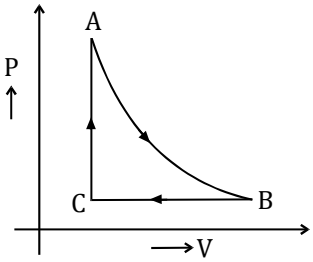
6. An ideal gas expands isothermally along AB and does 700 J of work.



(a) How much heat does the gas exchange along AB.

(b) The gas expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhusts 100 J of heat to its surroundings. How much work is done on the gas along this path.

7. For an ideal gas at 300 K occupies a volume of 0.36 m^3 at a pressure of 2 atm. The gas expands adiabatically until its volume becomes 1.44 m^3. Next gas is compressed isobarically to its original volume. Finally, the pressure is increased isochorically until the gas returns to its initial state. (see figure).

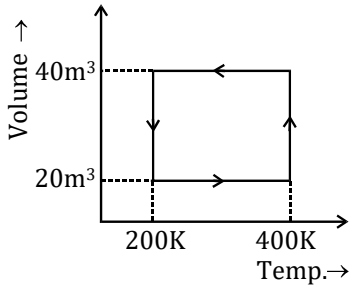


(a) Determine the temp. at the point B and C

(b) The work done during the process A to B

(Assume $\gamma = 1.5$, $R = 8\ \frac{J}{mol-K}$)

8. Following figure is given for 1 mole gas.



Then find out net work done in the cyclic process.

Second Law of Thermodynamics

Draw back of FLOT :

- No information about what part of heat converted into mechanical-work.
- Does not give proper direction of heat flow

Statement :

1. **Kelvin plank statement :** It state that in a cyclic process total heat can not be converted into mechanical work.
2. **Claussius statement :** It is impossible to have net heat flow from low temperature body to high temperature body.

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Heat Engine, Carnot Cycle, Refrigerators

Heat Engines

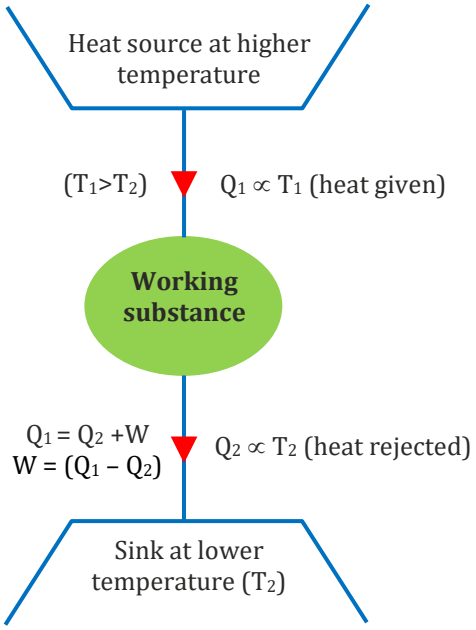
Efficiency of heat engine $\eta = \frac{\text{output}}{\text{input}}$

$\eta = \frac{\text{work}}{\text{heatgiven}}$

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

Maximum efficiency $\eta = \left(\frac{T_1 - T_2}{T_1}\right) \times 100\%$

$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\%$

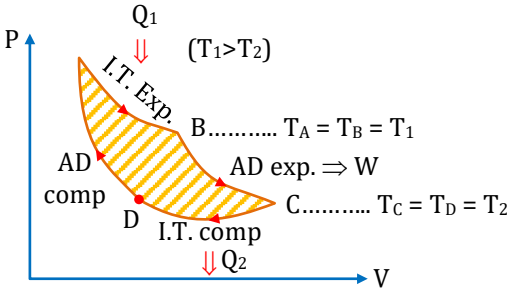

Heat source at higher temperature
(T1>T2)
Q1 ∝ T1 (heat given)
Working substance
Q1 = Q2 +W
W = (Q1 – Q2)
Q2 ∝ T2 (heat rejected)
Sink at lower temperature (T2)

Note : (1) If $T_1 = \infty$ K or $T_2 = 0$ K then $\eta = 100\%$ (That is practically impossible)
(2) Efficiency depends on temperature of source (T_1K) and temperature of sink (T_2 K)

Carnot Cycle

Enclosed area represent work done by the engine.

1. It is a hypothetical engine because working substance is an ideal gas.
2. It's efficiency is maximum but not 100%
3. This cycle process have two isothermal and two adiabatic process.



Efficiency of Carnot heat engine:

$$\eta = \frac{\text{output}}{\text{input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

$$\eta\% = \left[1 - \left(\frac{T_2}{T_1}\right)\right] \times 100\%$$

Illustration 118:

If a carnot engine works at a temperature between 300 K to 400 K, then find the efficiency of that engine.

Solution:

$$\eta\% = \left(\frac{T_1 - T_2}{T_1}\right) = \frac{400 - 300}{400} = \frac{100}{400} = \frac{1}{4} \text{ or } 25\%$$

Illustration 119:

A diatomic ideal gas is used in a Carnot engine as the working substance. if during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

Solution:

$$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{1}{32}\right)^{\frac{7}{5} - 1} = \frac{1}{4} \Rightarrow \text{Efficiency } \eta = \left(1 - \frac{T_2}{T_1}\right) = 1 - \frac{1}{4} = 0.75$$

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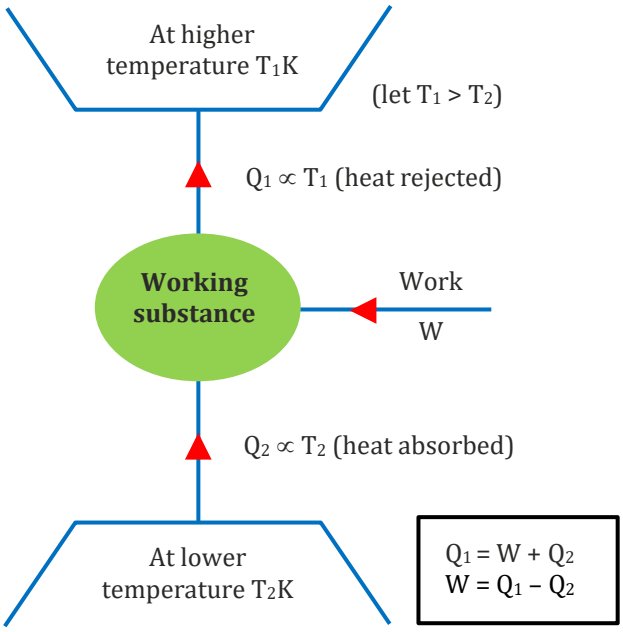
Refrigerators



In Carnot refrigerator heat is absorbed from a cold body & some external work is to be done on refrigerator and the total heat is given out at higher temperature.

- Refrigerator is just opposite to heat engine so (COP) of refrigerator is

$$\text{COP } (\beta) = \frac{\text{Heat absorbed}}{\text{Work}} = \frac{Q_2}{W} = \frac{Q_2}{(Q_1 - Q_2)}$$

$$\boxed{\text{COP}(\beta) = \left(\frac{T_2}{T_1 - T_2}\right)}$$

Relation between (η and β) $\boxed{\eta = \frac{1}{1+\beta}}$ or $\boxed{\beta = \frac{1}{\eta} - 1}$

Illustration 120:

Calculate COP of a Carnot refrigerator working between 30°C to 40°C.

Solution:

$$\beta = \frac{T_2}{T_1 - T_2} = \frac{303}{10} = 30.3$$

Illustration 121:

A Carnot engine has same efficiency between (i) 100 K and 500 K and (ii) TK and 900 K. Find the value of T?

Solution:

Efficiency $\eta = 1 - \frac{T_2}{T_1}$ or $\eta = 1 - \frac{100}{500} = 1 - \frac{T}{900}$ or $\frac{100}{500} = \frac{T}{900}$

$\therefore$ T = 180K

Illustration 122:

A Carnot engine takes 103 kilocalories of heat from a reservoir at 627°C and exhausts the remaining heat to a sink at 27°C. What will be the efficiency of the engine?

Solution:

$Q_1 = 10^3 \times 10^3$ cal = ($10^6 \times 4.2$ J)

$T_1 = 900$ K

$T_2 = 300$ K

$$\eta = \frac{T_1 - T_2}{T_1} \times 100 = \frac{600}{900} \times 100 = 67\%$$

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Illustration 123:

In the above problem, what will be the work performed by the engine?

Solution:

Work performed by the engine

$W = \eta Q_1 = \frac{2}{3} \times 10^6 \times 4.2$ or $W = 2.8 \times 10^6$ joule

Illustration 124:

A Carnot engine absorbs an amount Q of heat from a reservoir at an absolute temperature T and rejects heat to a sink at a temperature of T/3. The amount of heat rejected is: -

Solution:

$\because \eta = 1 - \frac{T/3}{T} = \frac{W}{Q_1}$

Where Q_1 = heat absorbed, Q_2 = heat rejected

$\Rightarrow \quad 1 - \frac{T/3}{T} = \frac{W}{Q_1} \quad \Rightarrow \quad \frac{2}{3} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$\Rightarrow \quad \frac{2}{3} = 1 - \frac{Q_2}{Q_1} \quad \Rightarrow \quad \frac{Q_2}{Q_1} = \frac{1}{3} \quad \Rightarrow \quad Q_2 = \frac{Q}{3}$

Illustration 125:

A Carnot engine having an efficiency of 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is: -

Solution:

$\eta = \frac{1}{10} \Rightarrow \beta = 1 - \frac{1}{\eta} \Rightarrow \beta = 9 = \frac{Q_2}{W} \Rightarrow Q_2 = 9 \times 10$

$Q_2 = 90J$ Heat absorbed at lower temp.

$Q_1 = 90 + 10 = 100$ J Heat released at higher temp.

BEGINNER'S BOX-9

1. An ideal engine which works between 27°C & 227°C takes 100 calorie of heat in one cycle. Calculate the work done in one cycle?
2. The coefficient of performance of a carnot refrigerator is 5/3. If the refrigerator operating at a intake temperature – 23°C then what will be the exhaust temperature?
3. A carnot engine absorbs 1000 J of heat from a reservoir at 127°C and reject 600 J of heat during each cycle.
   Calculate (a) The efficiency of engine
   (b) The temperature of the sink
   (c) The amount of the useful work done during each cycle.
4. A carnot engine, efficiency 40% and temperature of sink 300 K to increase efficiency up to 60% calculate change in temperature of source?
5. An ideal refrigerator is working between temperature 27°C and 127°C. If it expells 120 calorie of heat in one second then calculate its wattage?

BEGINNER'S BOX — ANSWERS KEY

BEGINNER'S BOX-1

1. 50 K < 50°F < 50°C
2. (a) All tie (b) 50°X, 50°Y, 50°W.
3. – 40°C or – 40°F.
4. (B)
5. 1.44×10^{-2} cm
6. In bimetallic strips the two metals have different thermal expansion coefficient. Hence on heating it bents towards the metal with lower thermal expansion coefficient.

BEGINNER'S BOX-2

1. 191.11 Cal. **2.** 50 kcal **3.** 1 cal **4.** 10° C
5. 1 : 1 **6.** 615 kcal. **7.** 4000 cal **8.** 160°C
9. 7.5°C, water **10.** (A)

BEGINNER'S BOX-3

1. (a) The thermal conductivity of brass is high, i.e., brass is a good conductor of heat. So, when a brass tumbler is touched, heat quickly flows from human body to tumbler. Consequently, the tumbler appears colder. On the other hand, wood is a bad conductor. So, heat does not flow from the human body to the wooden tray in this case. Thus, it appears comparatively hotter.

(b) This is because the two thin cloth enlose a layer of air between them. Since air is a bad conductor of heat therefore the conduction of heat is prevented.

(c) Because mudhouses are bad conductor of heat.

(d) Because air is trapped between their wings since air is bad conductor of heat therefore the conduction of heat is prevented.

(e) This is because wool contains air in its pores. Air is a bad conductor of heat. In fact, wool is also a bad conductor of heat so both the air and wool donot permit heat to be conducted away from. So woolen clothes are warm.

2. (a) 240 W (b) 1.5 **3.** 40°C **4.** $\theta_1 = 116°C, \theta_2 = 74°C$
5. 68°C; 0.9 °C/cm **6.** 7 : 2

BEGINNER'S BOX-4

1. (a) A body whose reflectivity is large would naturally absorb less heat. So, a body with large reflectivity is a poor absorber of heat. Poor absorbers are poor emitters. So, a body with large reflectivity is a poor emitter.

(b) Let T be the temperature of the hot iron in the furnace. Heat radiated per second per unit area, $E = \sigma T^4$

When the body is placed in the open at temperature T_0, then the heat radiated/second/area, $E' = \sigma (T^4 - T_0^4)$

Clearly E' < E. So, the optical pyrometer gives too low a value for the temperature in the open.

(c) The lower layers of earth's atmosphere reflect infrared radiations from earth back to the surface of earth. Thus, the heat radiation received by the earth from the sun during the day are kept trapped by the atmosphere. If atmosphere of earth were not there, its surface would become too cold to live.

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(d) When the electric heater is switched on, a stage is quickly reached when the rate at which heat is generated by electric. Current becomes equal to the rate at which heat is lost by radiation. This is case of thermal equilibrium.

(e) Because black surface is good absorber of heat.

(f) On a clear night, the earth radiates energy into space at a rate proportional to the fourth power of its temperature (about 300K). The incoming radiation from space is very small because its average temperature is nearly absolute zero. On the other hand with cloud over, the earth radiates at 300K, but the radiation is absorbed in the clouds, which radiate energy back to earth again the radiation is trapped, like the green house effect.

(g) This is due to the fact that a vaccum is created between the two walls of the thermos flask. Heat can neither flow from inside the flask to outside nor from outside air to the liquid inside the flask. In this way, loss of heat by conduction and convection has been minimised. To minimise loss of heat by radiation the surface is made shining.

2. $\sqrt{\frac{r_2}{r_1}}$ **3.** 40 °C **4.** 28 °C **5.** 1927 K

BEGINNER'S BOX-5

1. $\frac{1}{2}$ mole **2.** 120 kPa **3.** 2.2×10^{24} **4.** (a) $\frac{N_1}{N_2} = \frac{3}{2}$; (b) 0.947

BEGINNER'S BOX-6

1. (a) 6.21×10^{-21} J; (b) 10.35×10^{-21} J; (c) 6231J **2.** (a) 127°C, (b) 527°C

3. (a) 3, (b) $\frac{5}{3}$, (c) 450R

4. $v_{avg} = \frac{v_1 + v_2 + v_3}{3}$,

Root mean square speed = $\sqrt{\frac{v_1^2 + v_2^2 + v_3^2}{3}}$

5. 9.2 m/s **6.** (a) $\frac{E_1}{E_2} = \frac{1}{1}$ ($\because$ T = same for both); (b) $v_{rms} = \sqrt{\frac{70.9}{39.9}} = \frac{1.33}{1}$

BEGINNER'S BOX-7

1. (a) $W_{ABCA} = \frac{1}{2} \times (2V_0 - V_0) \times (3P_0 - P_0) = P_0V_0$ (b) $\frac{5}{2} P_0V_0$ and $3P_0V_0$, (c) $\frac{P_0V_0}{2}$

2. -5×10^5 J

3. $W_{AB} = 6000$ J

W_{BC} = zero

$W_{CD} = 1000$ J

W_{DA} = zero

4. (a) 24×10^2 J; (b) 36×10^2 J; (c) 60×10^2 J

5. 4.16 J/cal

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BEGINNER'S BOX-8

1. 3650 J

2. (a) 162.5 cal; (b) 237.5 cal

3. (A) – p, (B) – q , (C) – r

4. (A) – q , (B) – r , (C) – p , (D) – s

5. (A) –p, s, (B) –p, q, r, s, (C) –s ,(D) –q, s, (E) –r, s

6. (a) 700 J; (b) 500 J

7. (a) $T_B = 150$ K ; $T_C = 37.5$ K; (b) 72×10^3 J

8. 1152.6 joule.

BEGINNER'S BOX-9

1. 168 J

2. 400 K

3. (a) 40%; (b) 240K; (c) 400 J

4. 250 K

5. 126 watt.